CPM and PERT

TLH

Critical Path Method

Basics

Activity and events

Activities consume resources (time , money, raw materials,…)

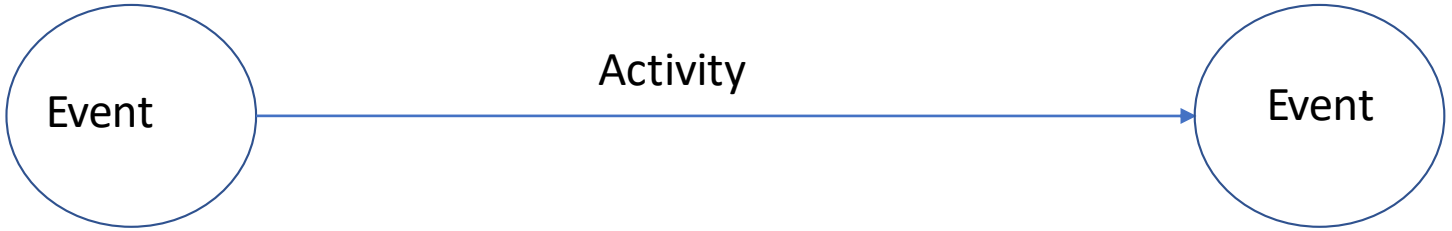
Events are point in time and do not consume resources.

Activity and events

Basic rules

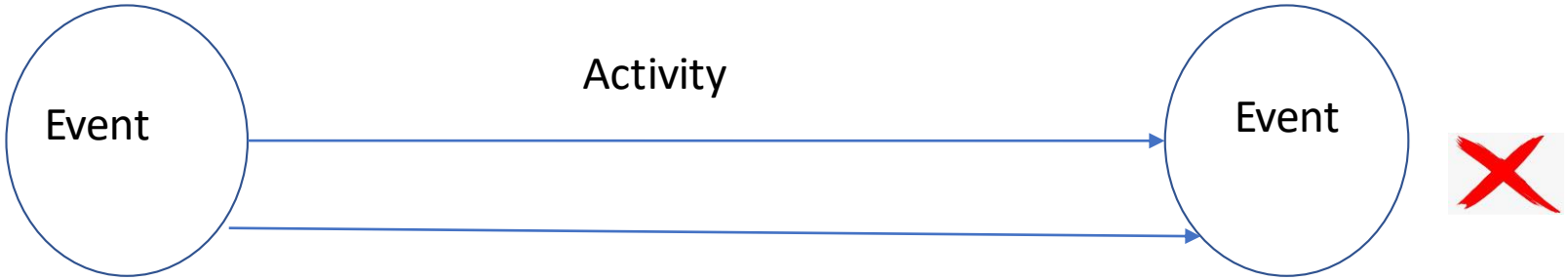
Only one start and only one end for a project.

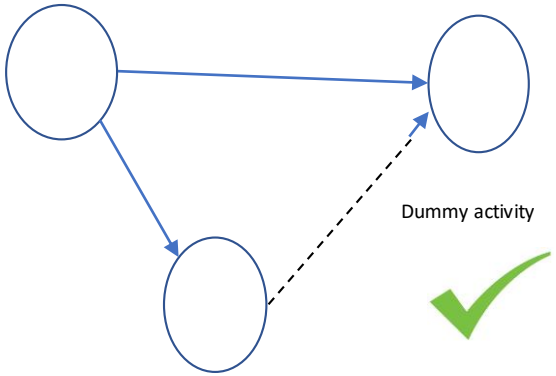
Not more than one activity between any two events.

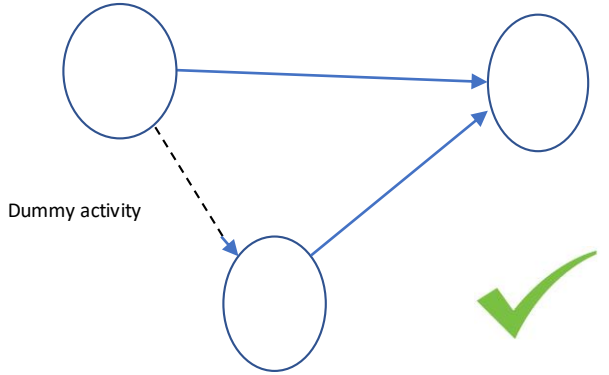


Basic rules

Not more than one activity between any two events.







Drawing a network

Task/ Activity	Precedence	Duration
A	-	2
B	A	3
C	A	3
D	C	4
E	D	8
F	B,E	6
G	F	2

How to draw a network?

Precedence	Task/ Activity	Successor	Duration
-	A		2
A	B		3
A	C		3
C	D		4
D	E		8
B,E	F		6
F	G		2

How to draw a network?

Precedence	Task/ Activity	Successor	Duration
-	A	B, C	2
A	B		3
A	C		3
C	D		4
D	E		8
B,E	F		6
F	G		2

How to draw a network?

Precedence	Task/ Activity	Successor	Duration
-	A	B, C	2
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A	C		3
C	D		4
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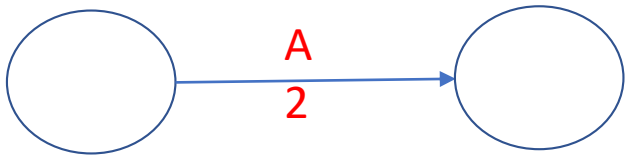
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2

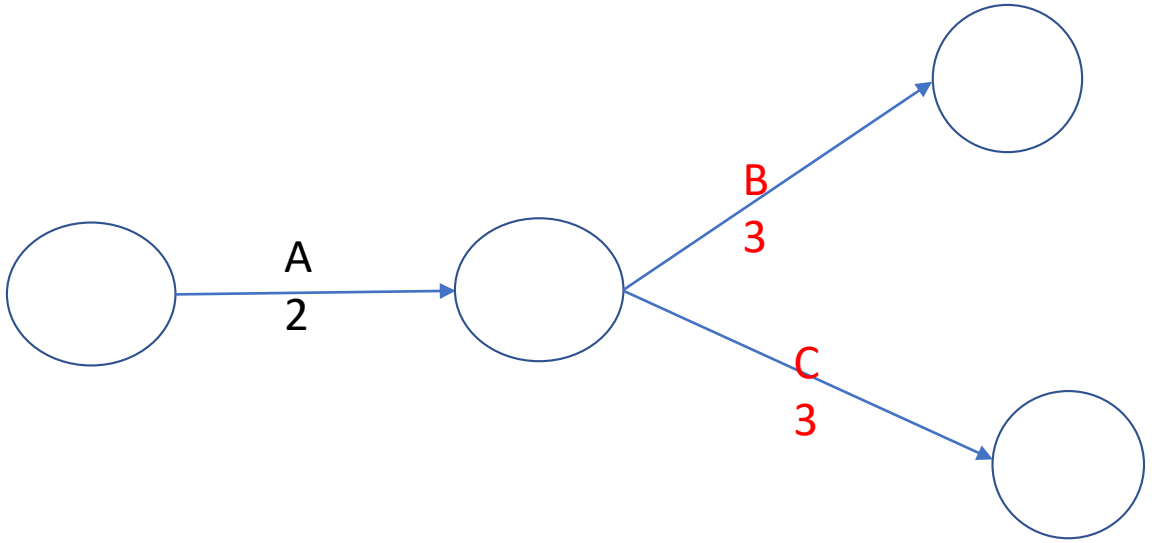
Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



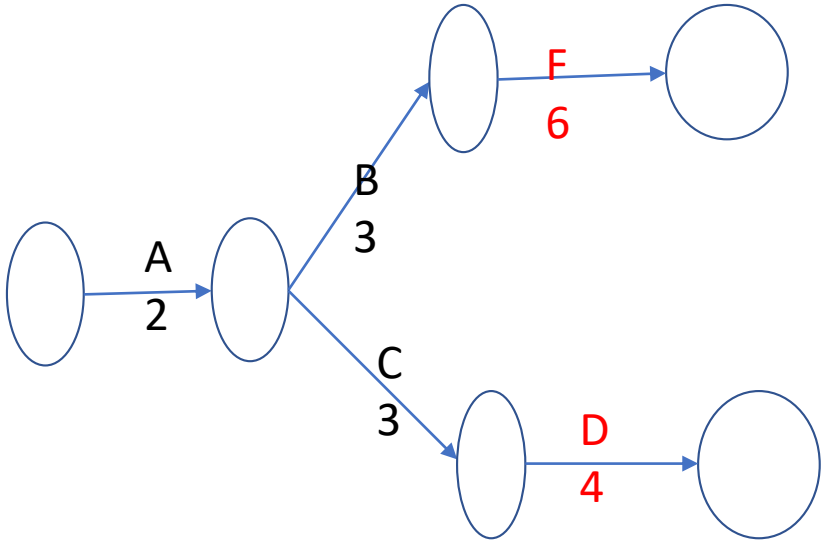
Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	**F**	3
A	C	**D**	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
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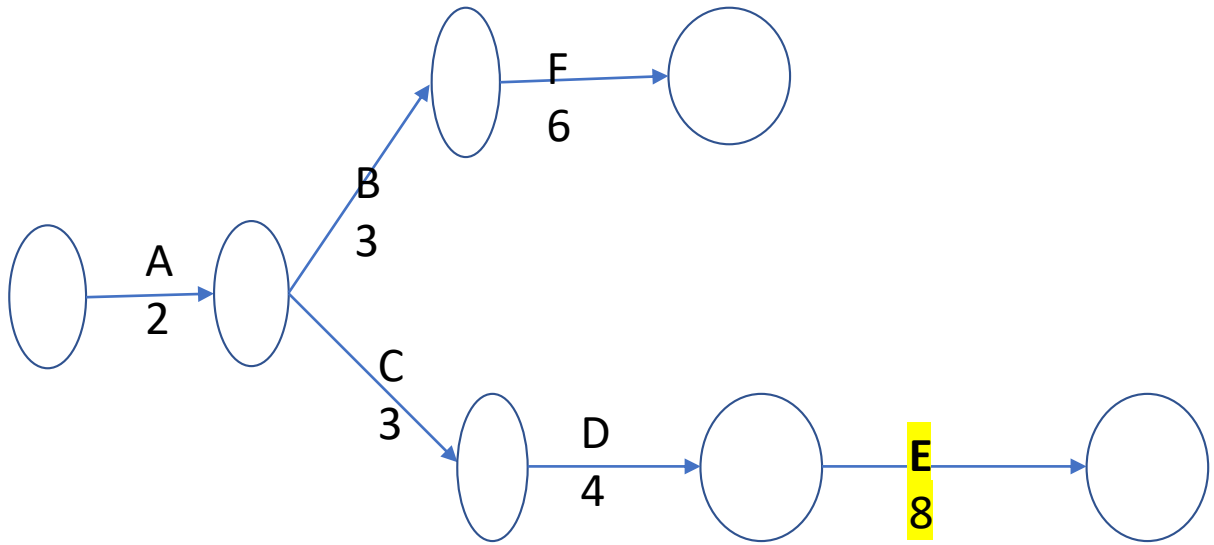
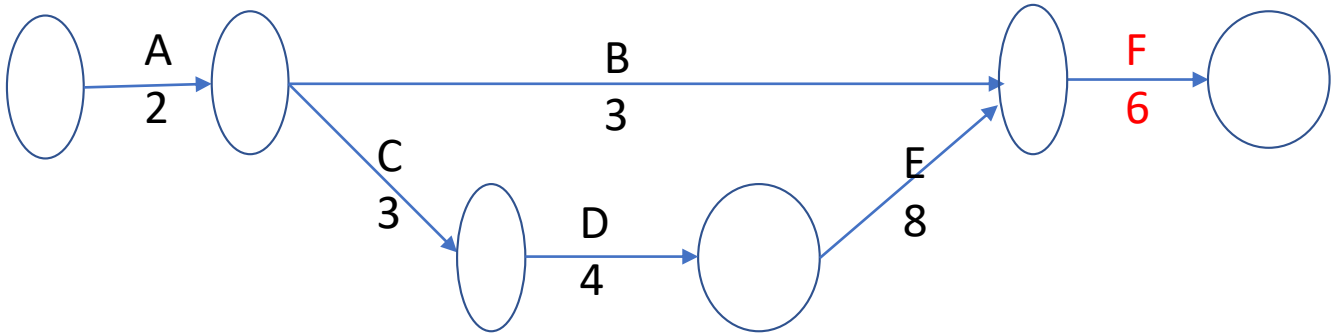


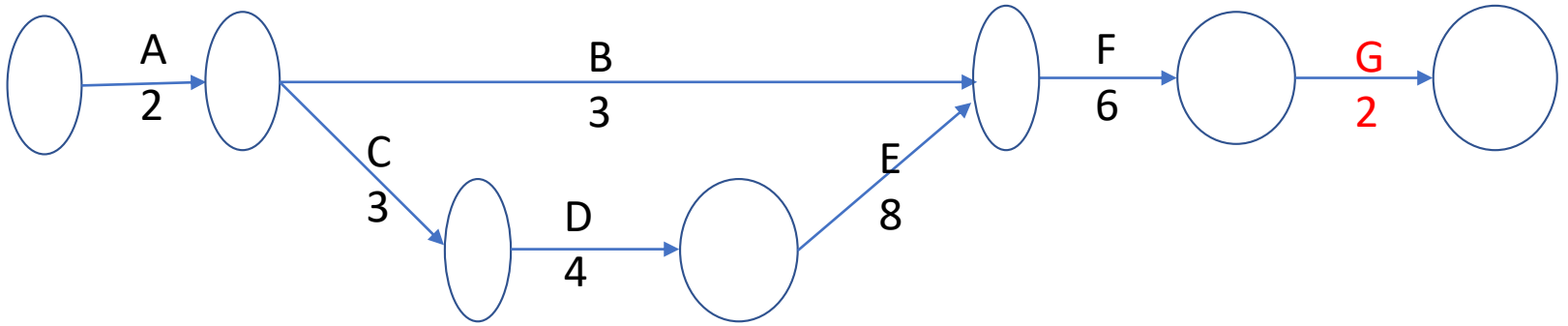
Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



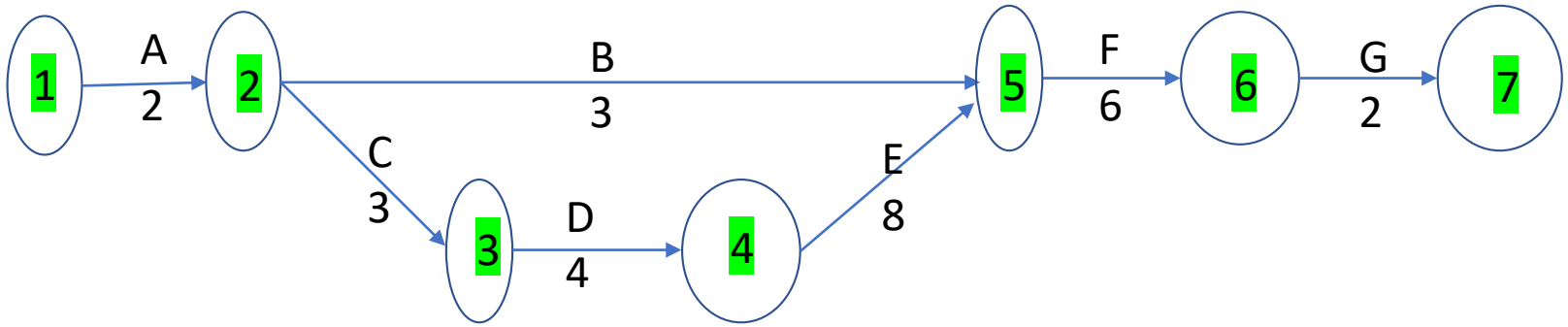
Here is the network diagram

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Numbering the events (Ford Fulkerson's rule)

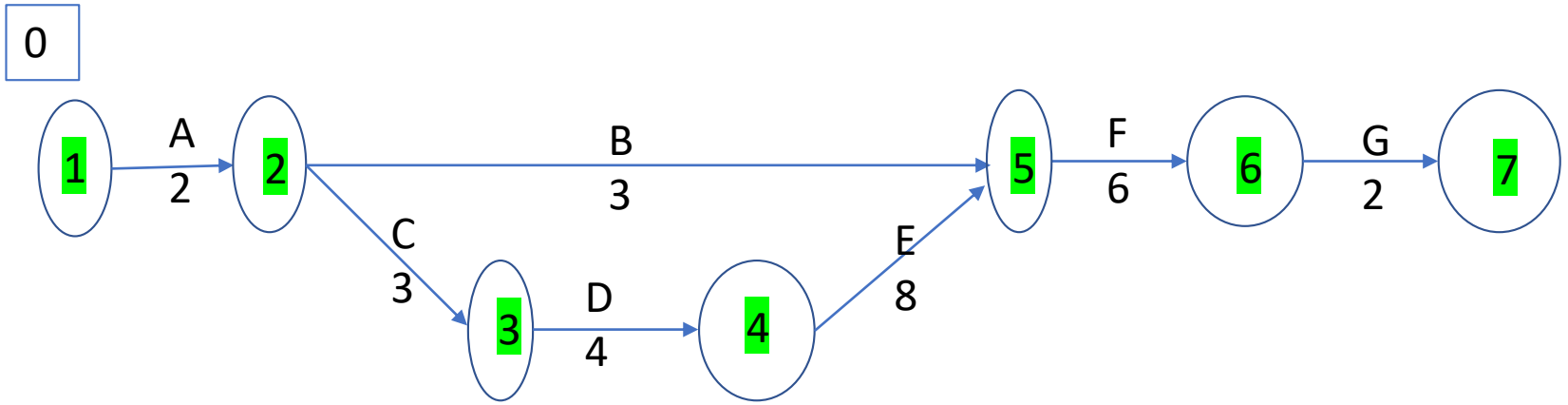
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Earliest start time of an activity

(EST of an activity = EST of the tail event)

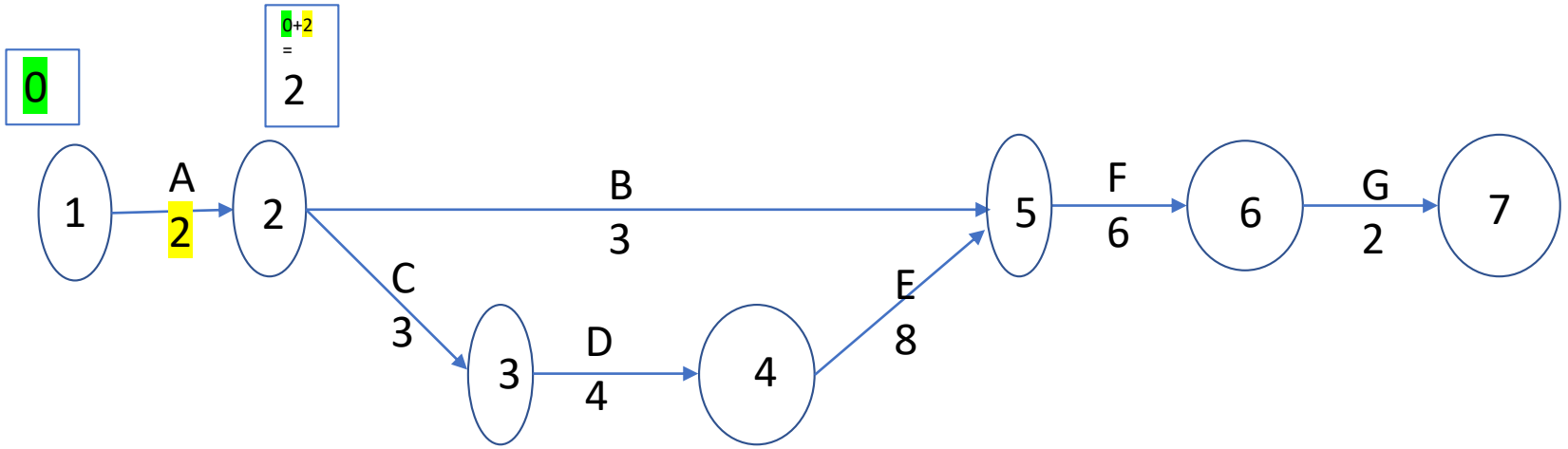
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



The Earliest Start of an activity A is the EST of tail event of A, that is event 1.
Always the start of a project is at time 0.

Earliest start time

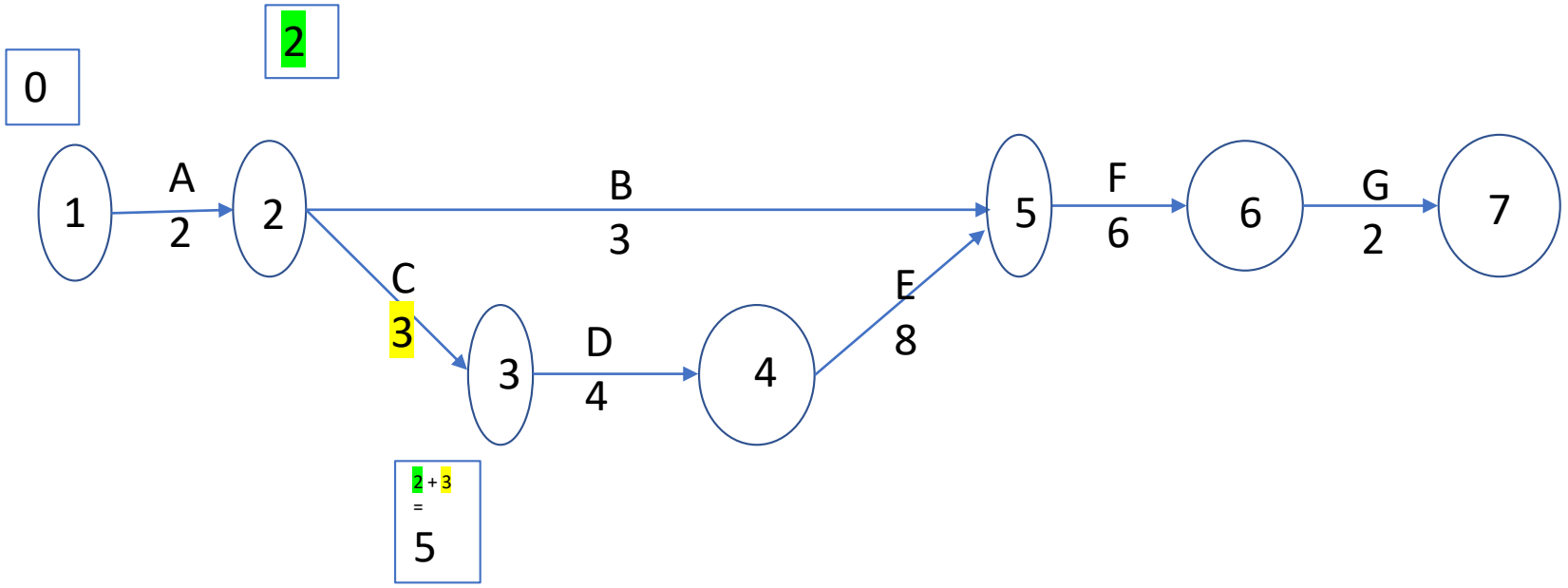
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



The Earliest Start of an activity C is the EST of tail event of C, that is event 2.
Start was at time 0 and activity A took 2 days.
Therefore, activity C (as well as B) cannot start before the end of 2nd day.
EST of activity C (as well as B) is 2

Earliest start time

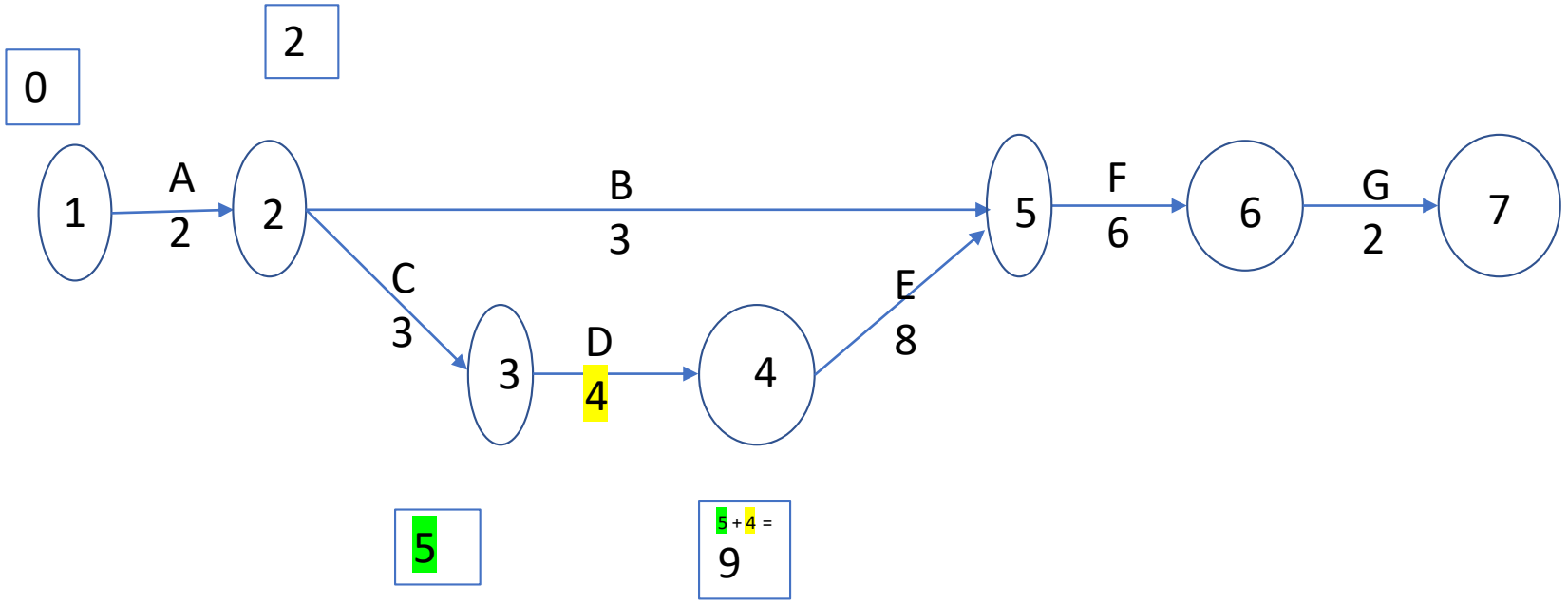
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



The Earliest Start of an activity D is the EST of tail event of D, that is event 3.
Start of activity C was at time 2 and activity C took 3 days. Therefore, activity D cannot start before the end of 5^{th} day.
EST of activity D is 5

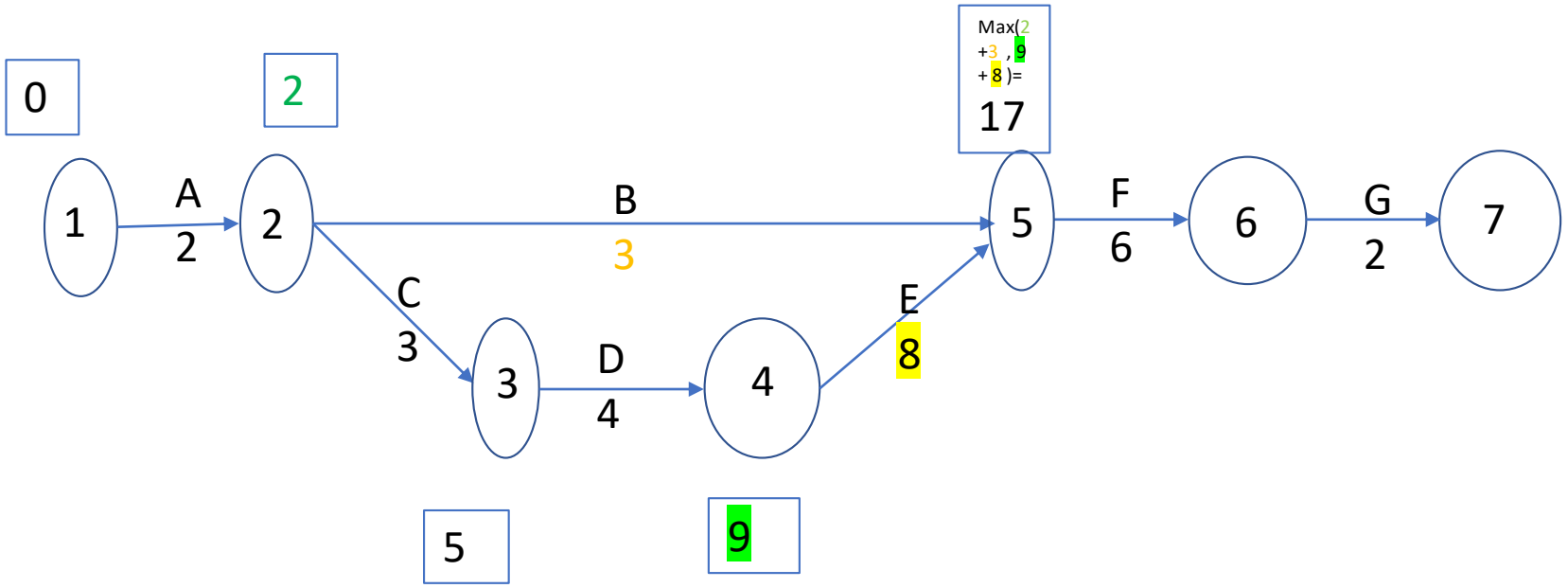
Earliest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Earliest start time

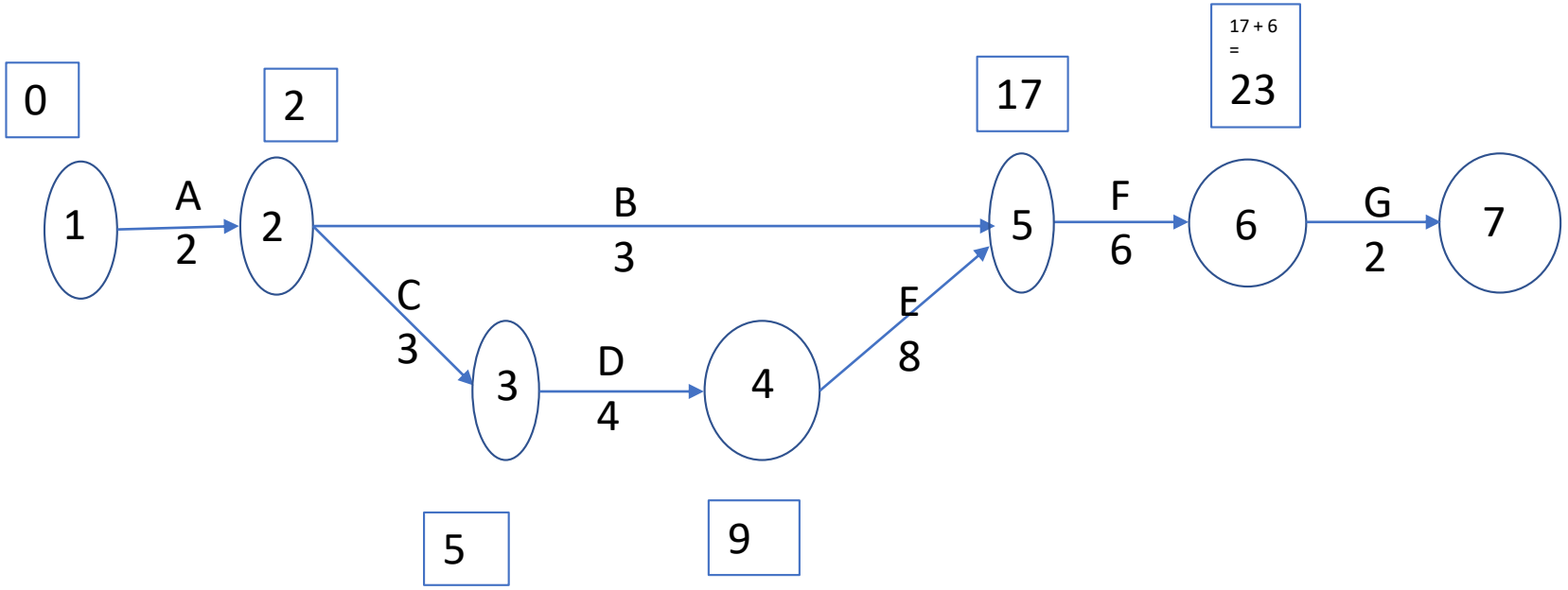
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



The Earliest Start of an activity F is the EST of tail event of F, that is event 5.
Activity B can be completed on (2+3=) 5th day. But activity E will cannot be completed before 17th day (9+8=17)
EST of activity F is 17.

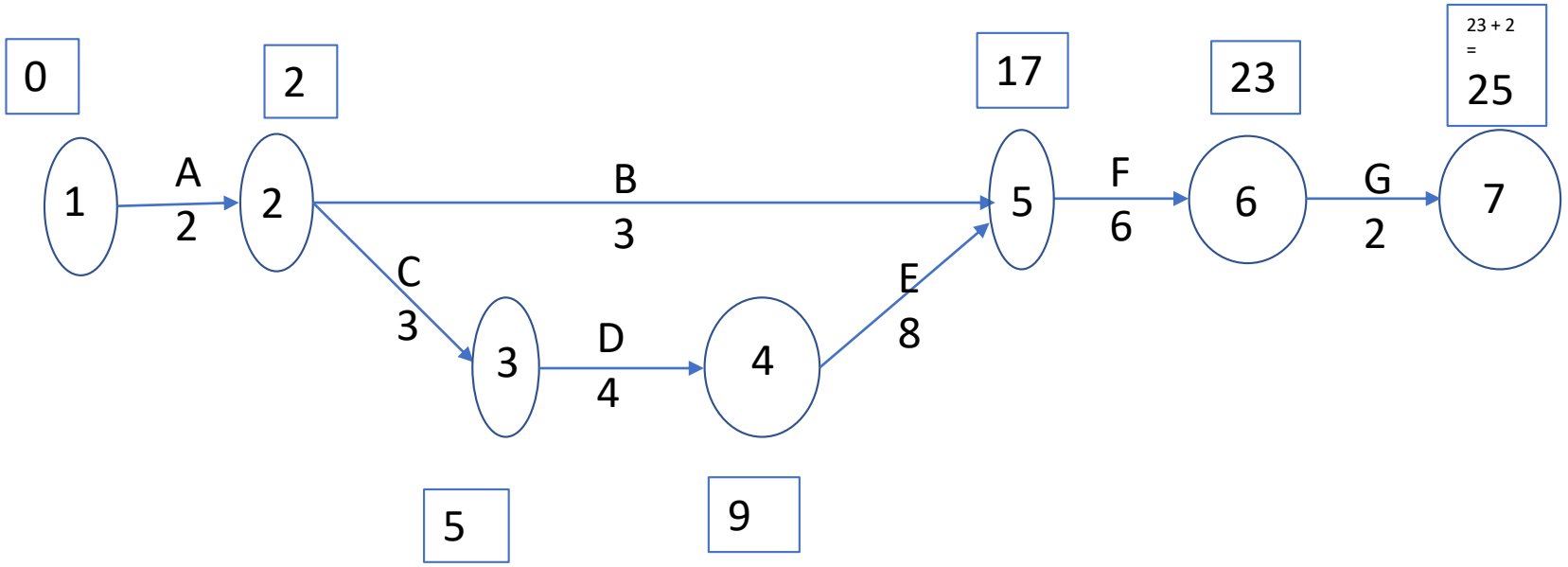
Earliest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Latest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2

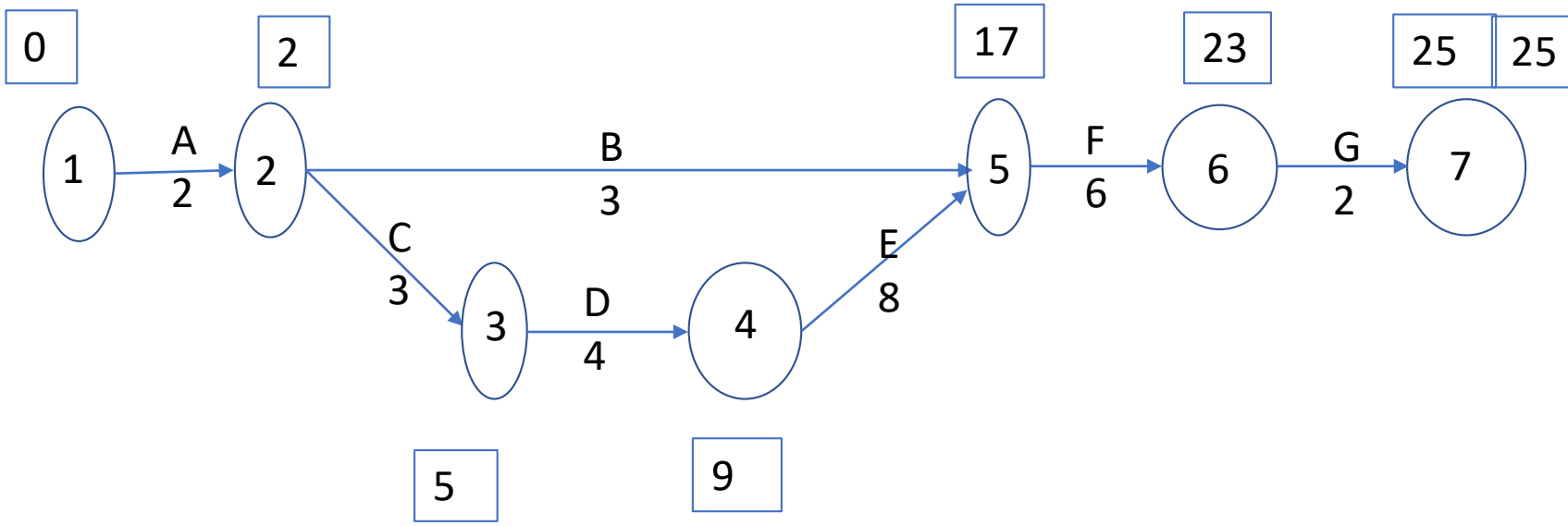


Clearly, the project will take a minimum of 25 days.

Latest start time of an Activity

(LST of an activity is the LST of the tail event of the activity)

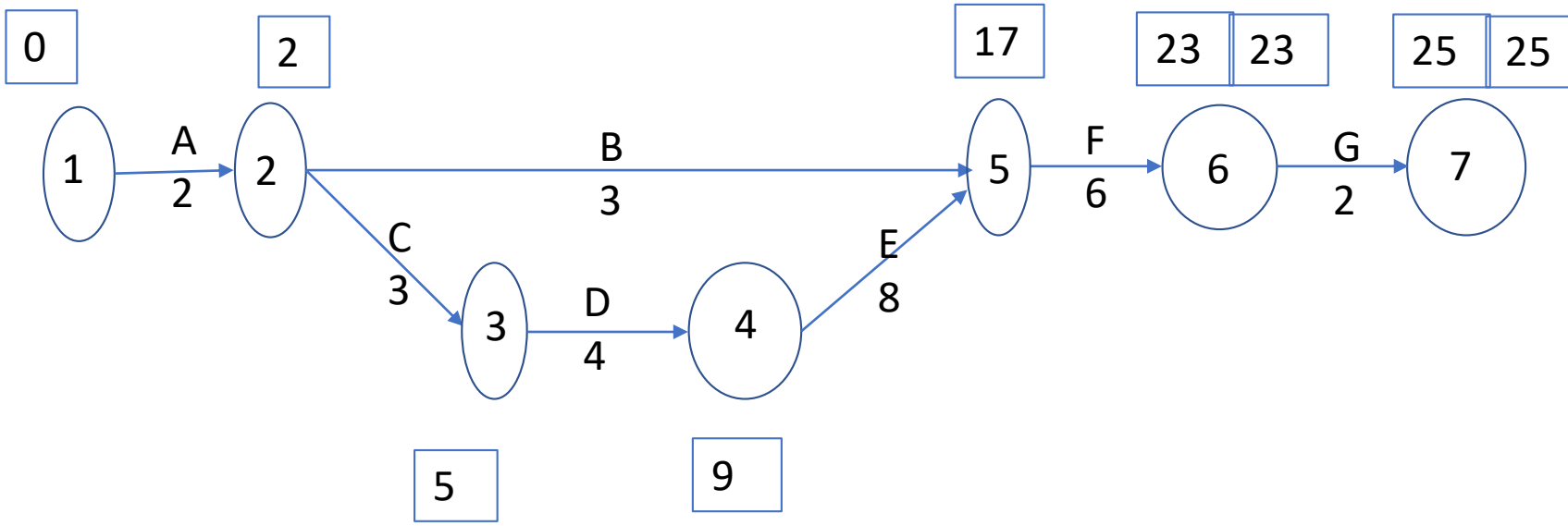
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



LST of event 7 is the same EST of event 7.

Latest start time

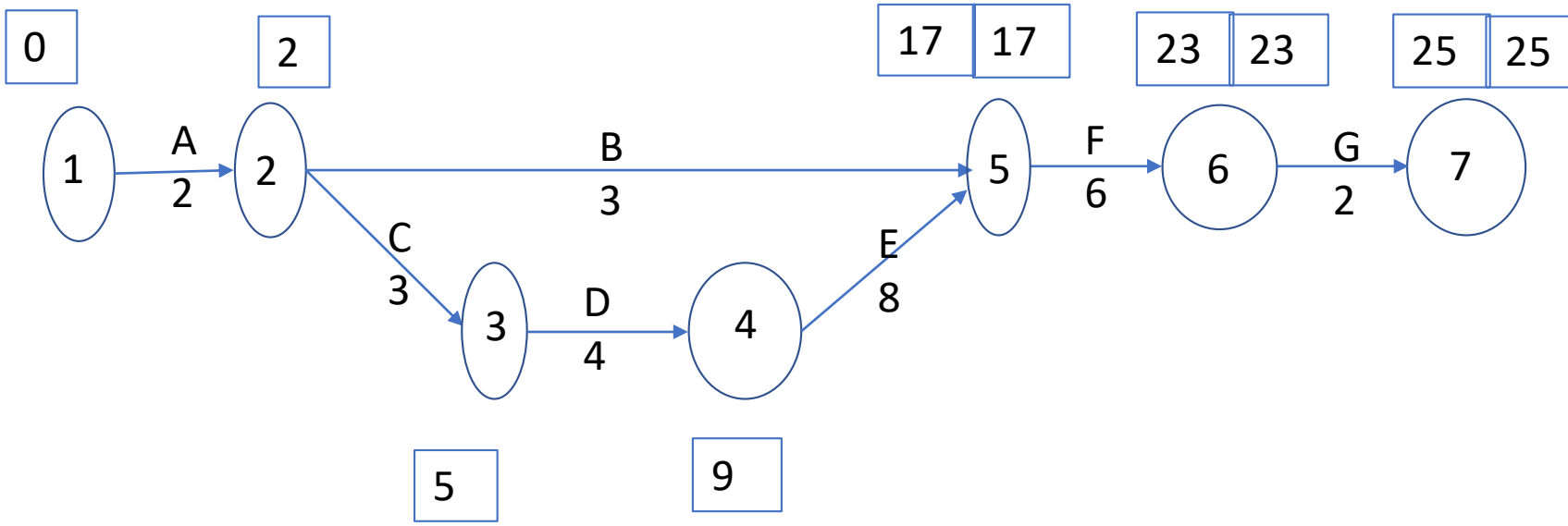
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



LST of activity G is LST of head event – duration of activity G

Latest start time

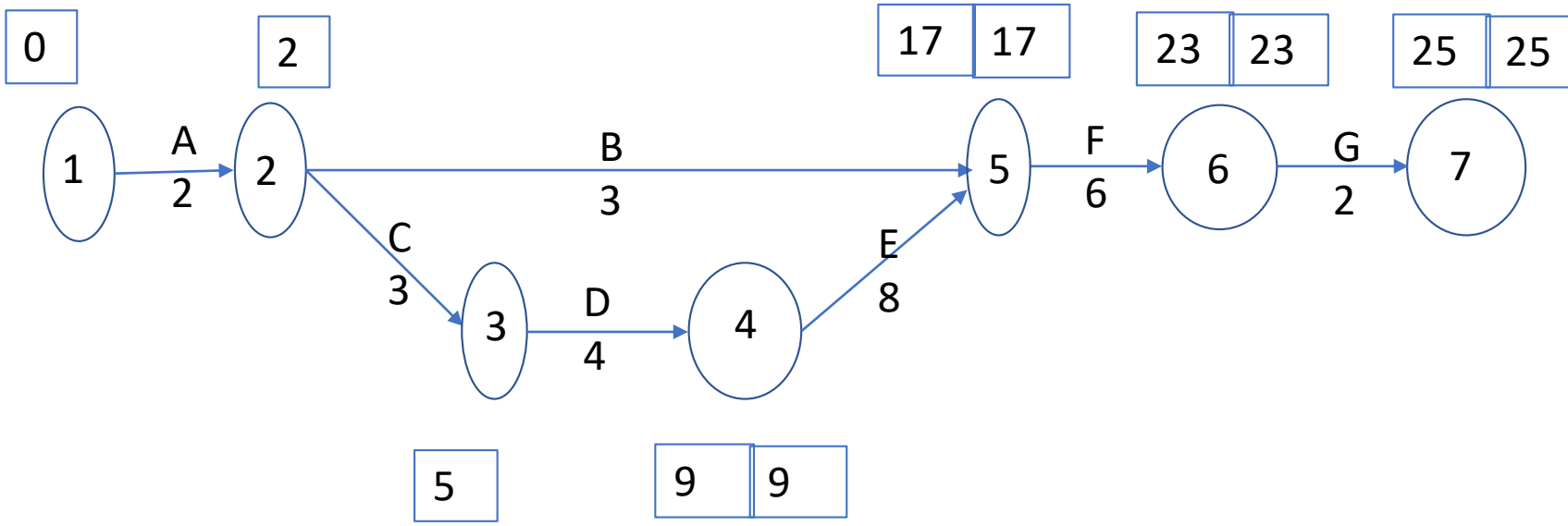
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



LST of activity F = LST of head event of F(that is event 6) – Duration of activity F

Latest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2

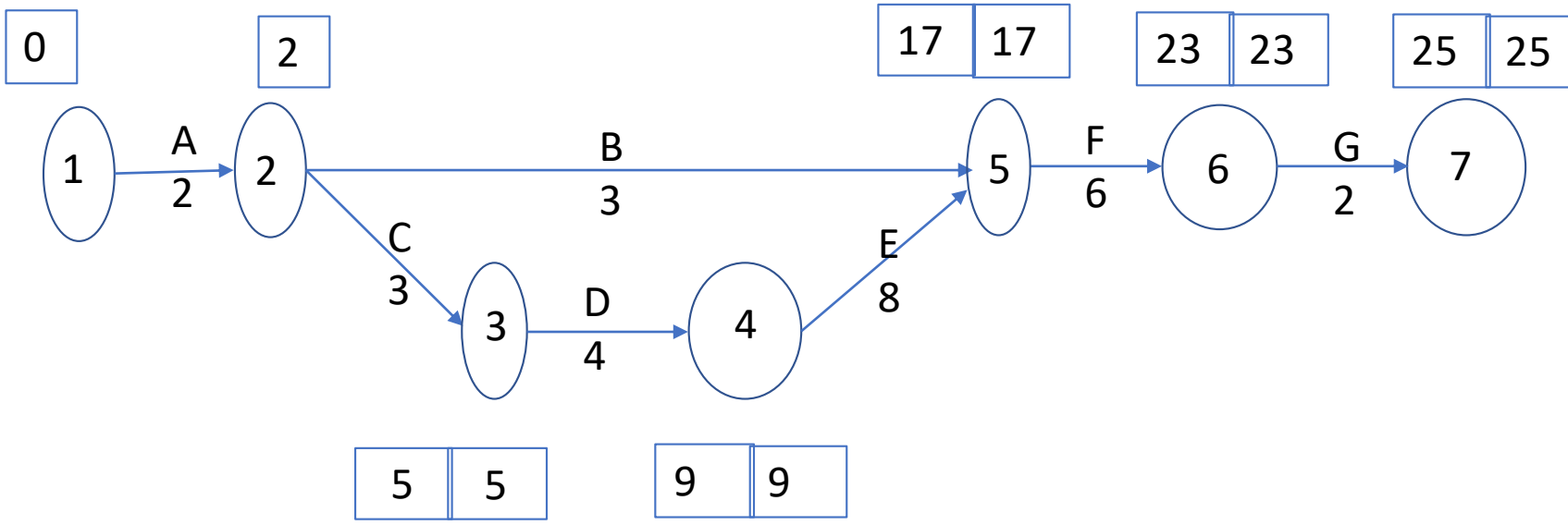


LST of activity E = LST of head event of E(that is event 5) – Duration of activity E

Quiz: Why are we doing it for activity E instead of activity B?

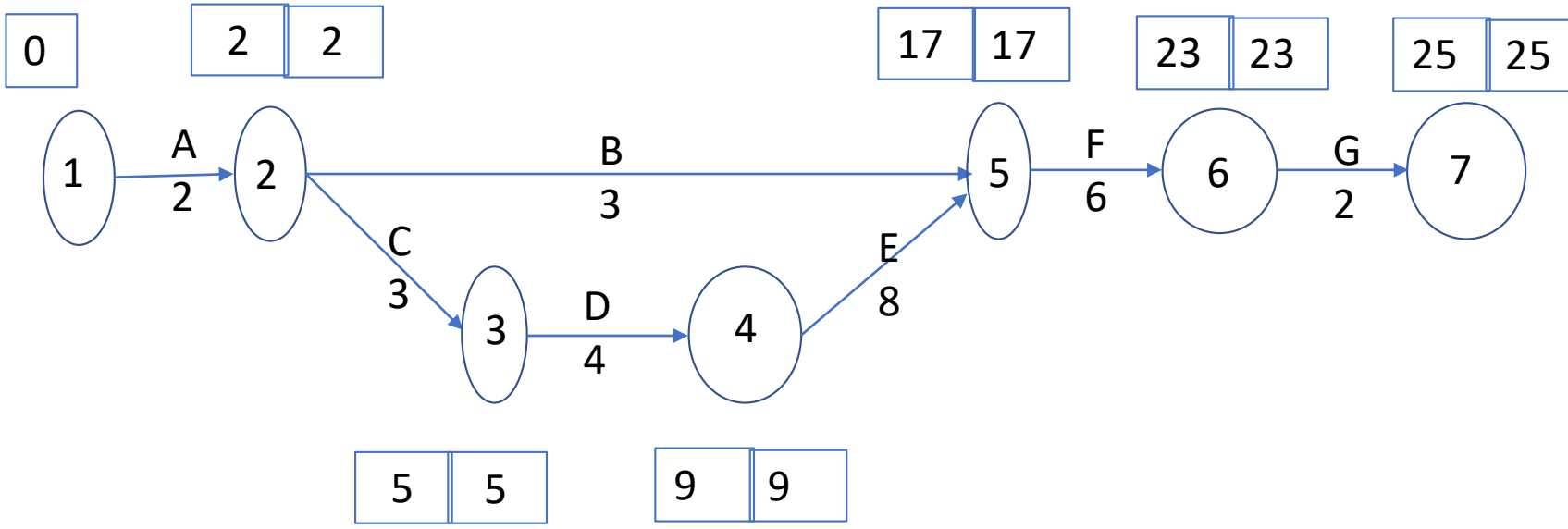
Latest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Latest start time

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2

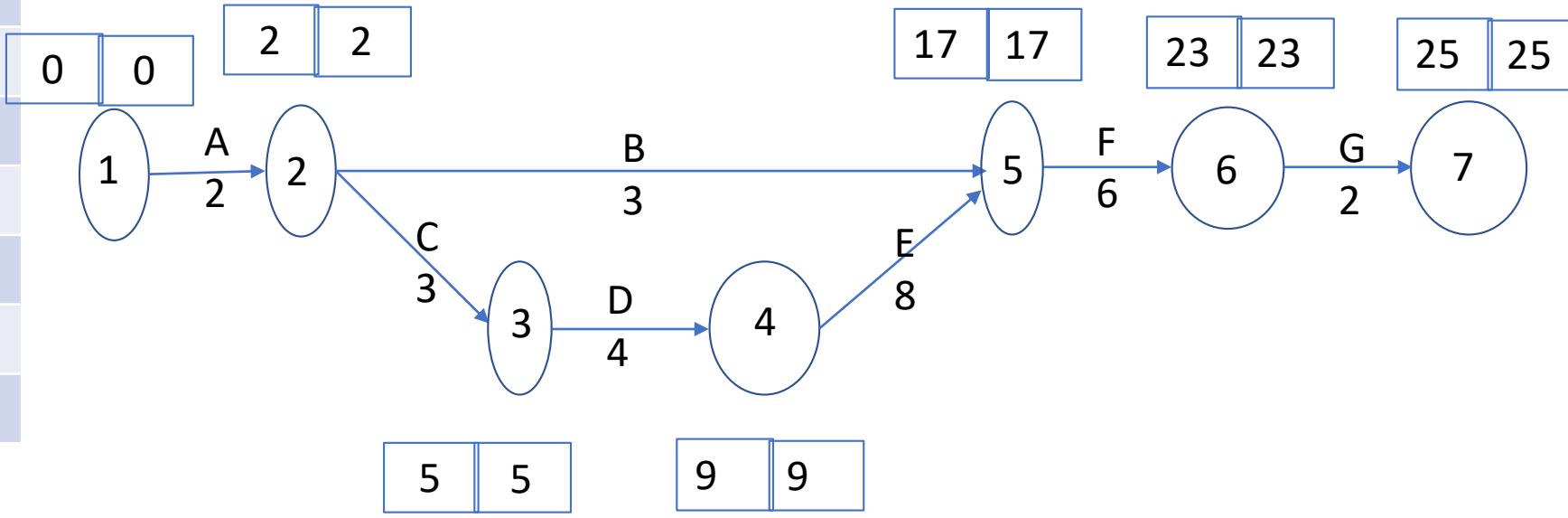


LST of activity C as well as B = Min{LST of head event of C(that is event 3) – Duration of activity C, LST of head event of B(that is event 5) – Duration of activity B}

Quiz: but why?

Latest Start Time

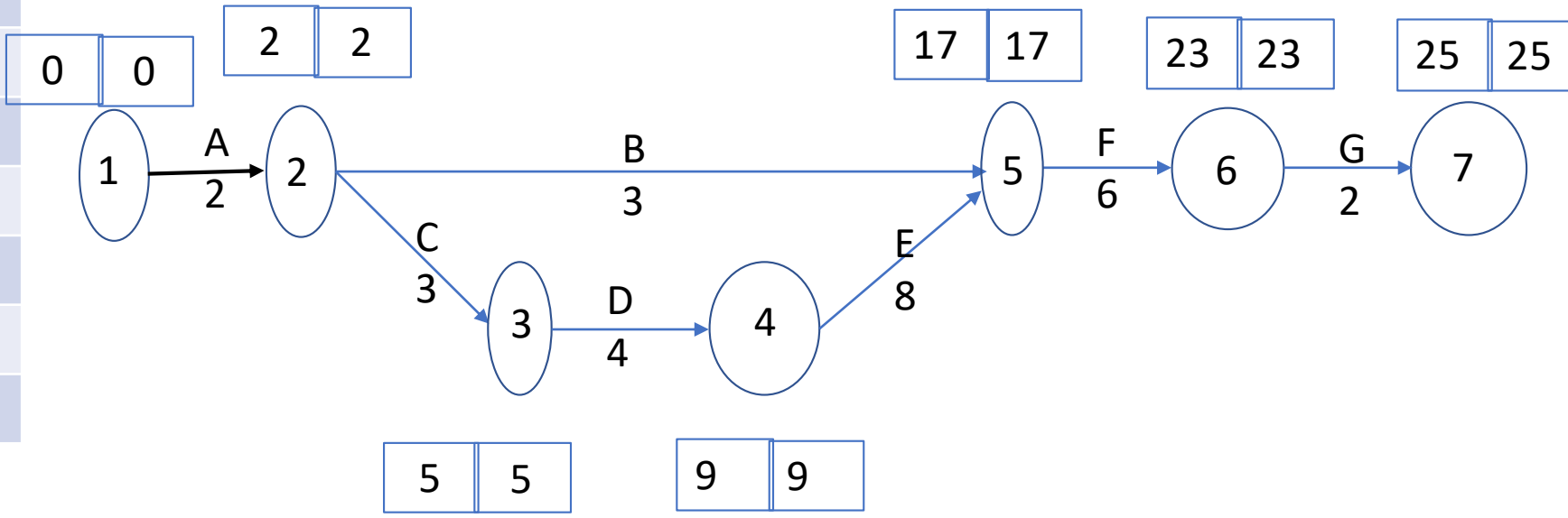
Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Quiz: if the LST of the first event be always Zero?

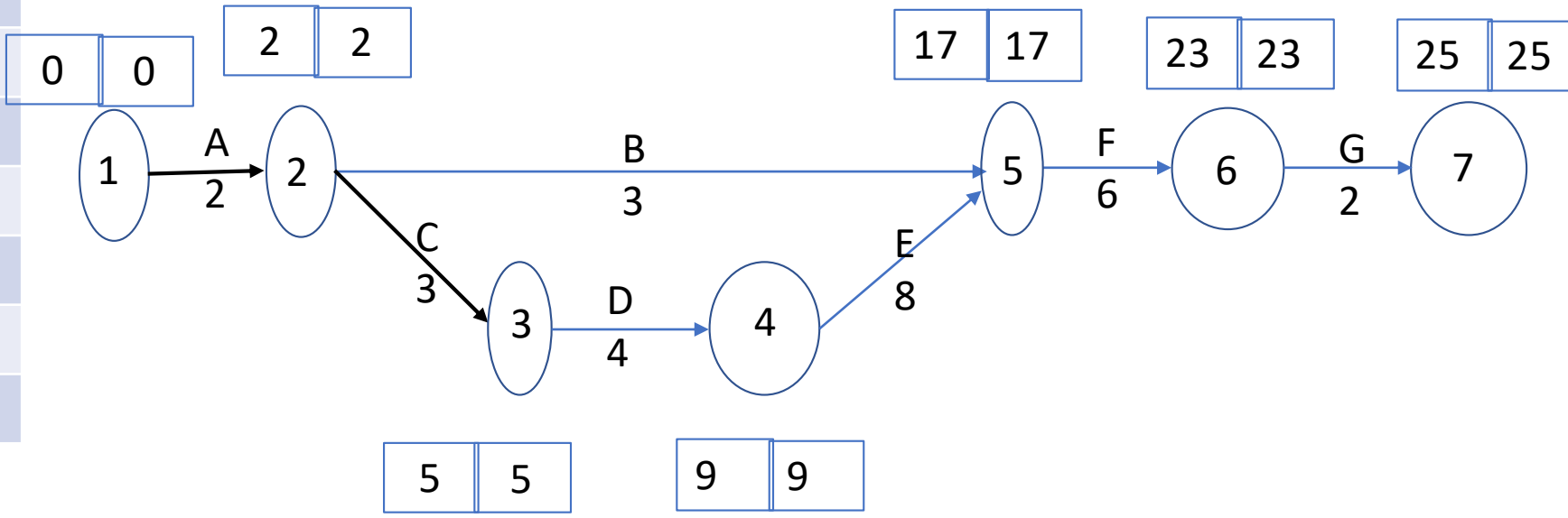
Finding the critical activities in an orderly fashion

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



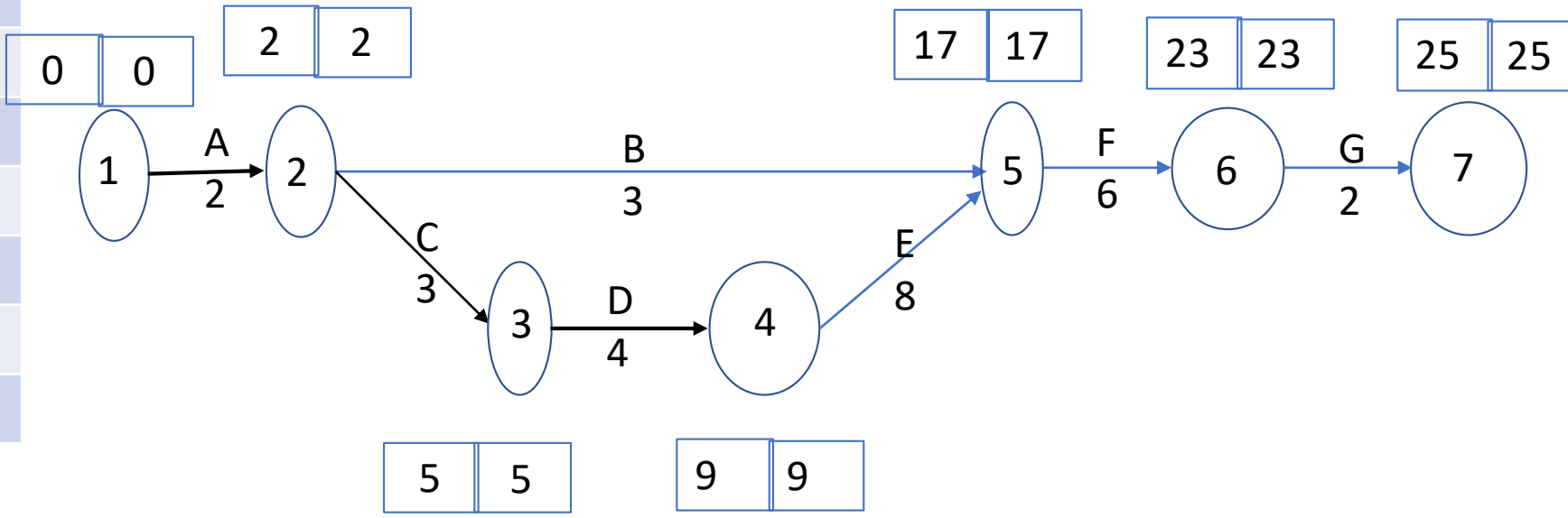
Finding the critical activities in order

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



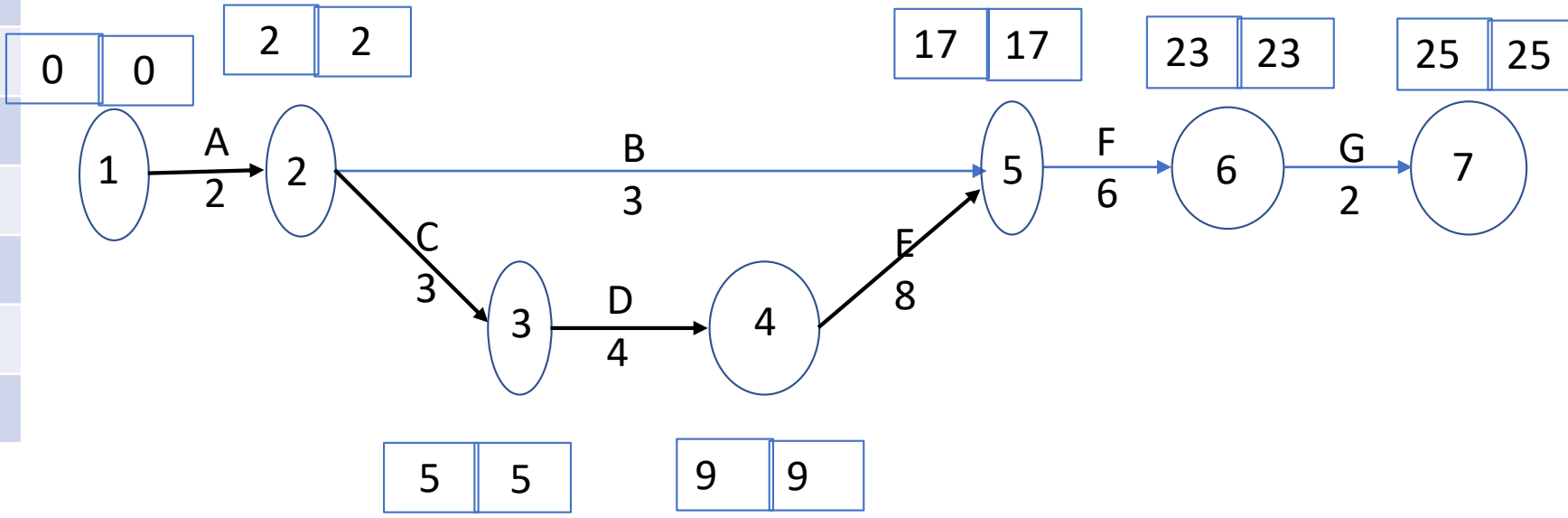
Finding the critical activities in order

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



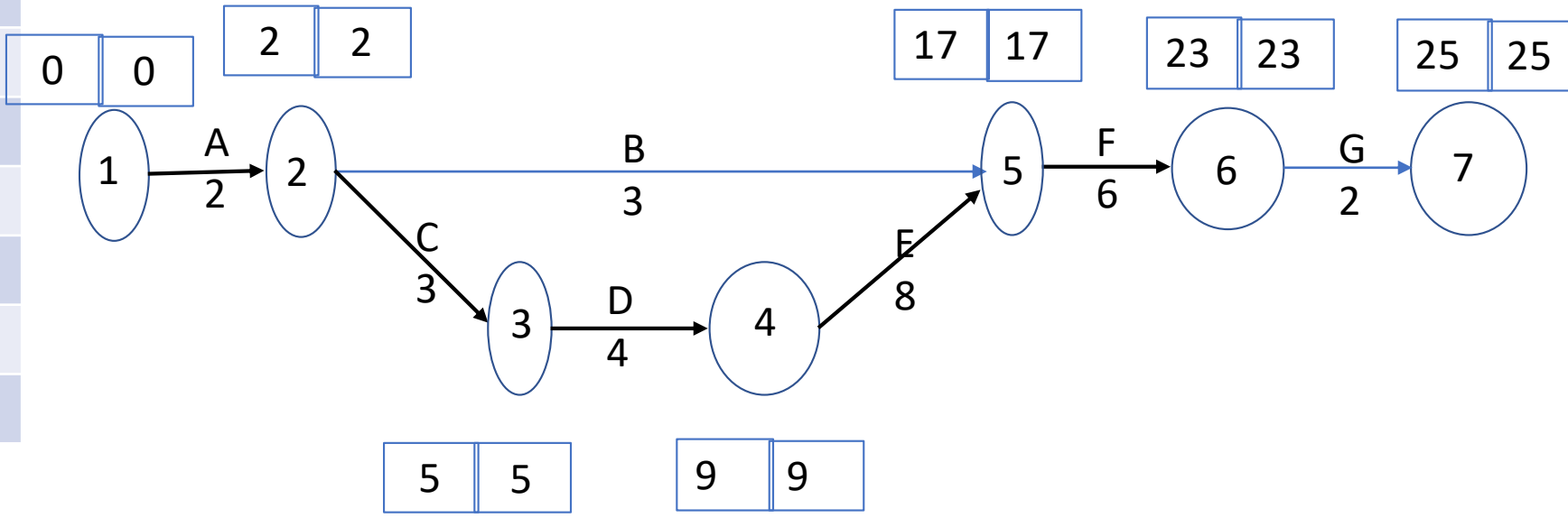
Finding the critical activities in order

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



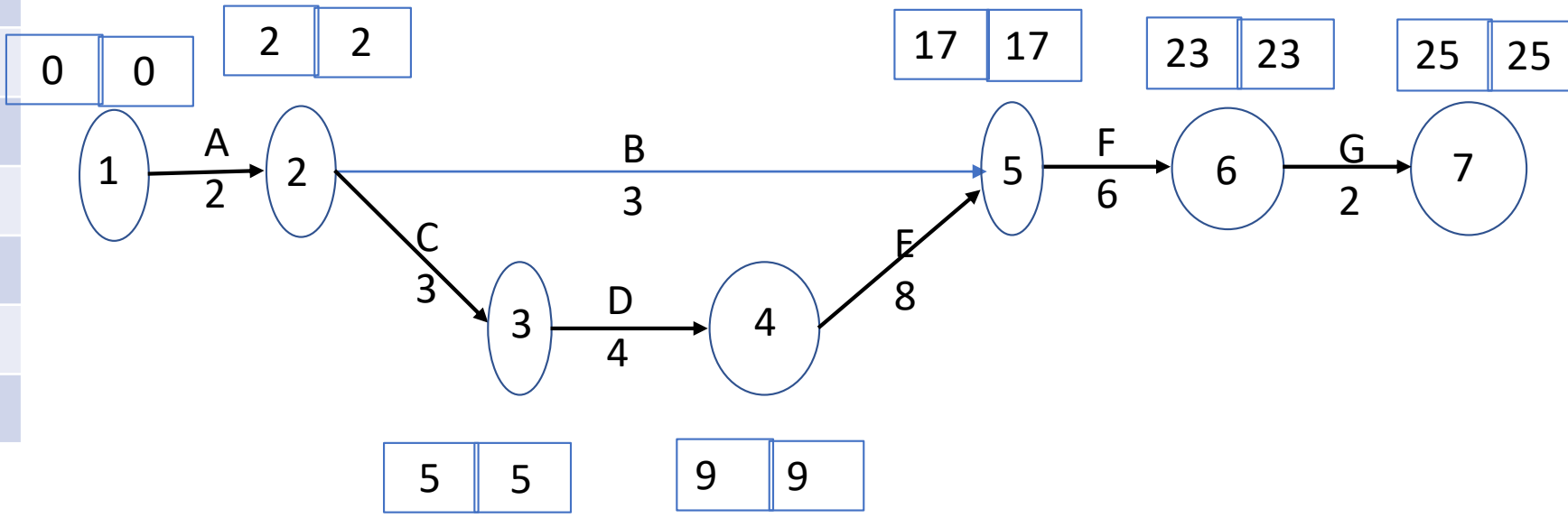
Finding the critical activities in order

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



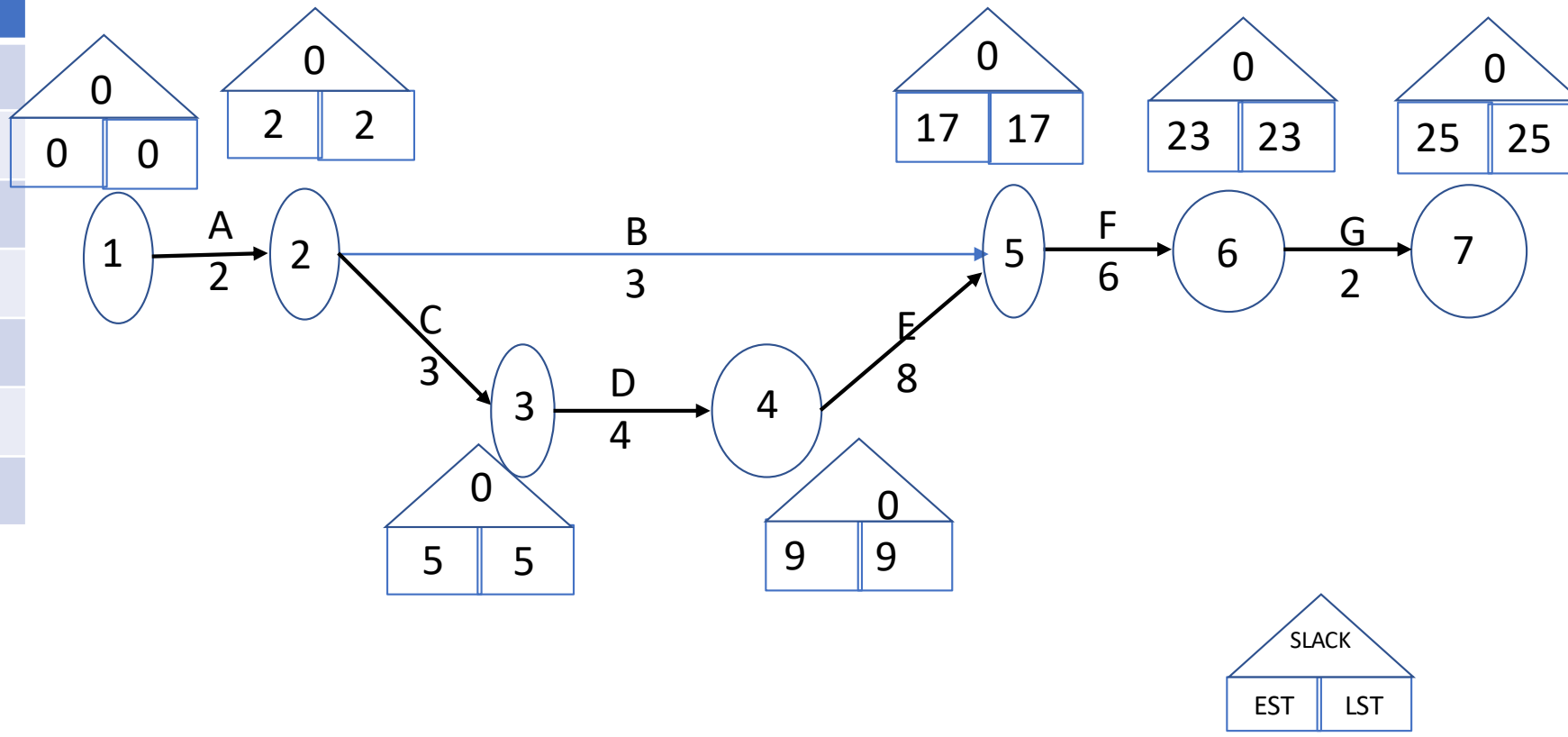
Finding the critical activities and Hence the critical path

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Finding the critical activities and Hence the critical path

Pre	Task	Succ	Duration
-	A	B, C	2
A	B	F	3
A	C	D	3
C	D	E	4
D	E	F	8
B,E	F	G	6
F	G	-	2



Results

The project duration is 25 days

The critical path is A-C-D-E-F-G

Any delay in these activities will delay the project.

Questions?

Which activity has free time? And by how many days?

Can we use Excel?

Can we use online tools?

Questions?

Which activity has free time? And by how many days?

Can we use Excel?

Can we use online tools?

Questions?

Which are activities that have free time? And by how many days?

Can we use Excel?

Can we use online tools?

Questions?

Is it available in EoC?

Questions

Can I use it for resource allocation and re-allocation?

Questions

Can I update it in the middle?

Questions

Can I do a crashing?

Questions

Can we use it for unfamiliar projects?

Program Evaluation and Review Technique (PERT)

Recap : Tail Event and Head Event of an Activity

Activities consume resources (time , money, raw materials,…)

Events are point in time and do not consume resources.

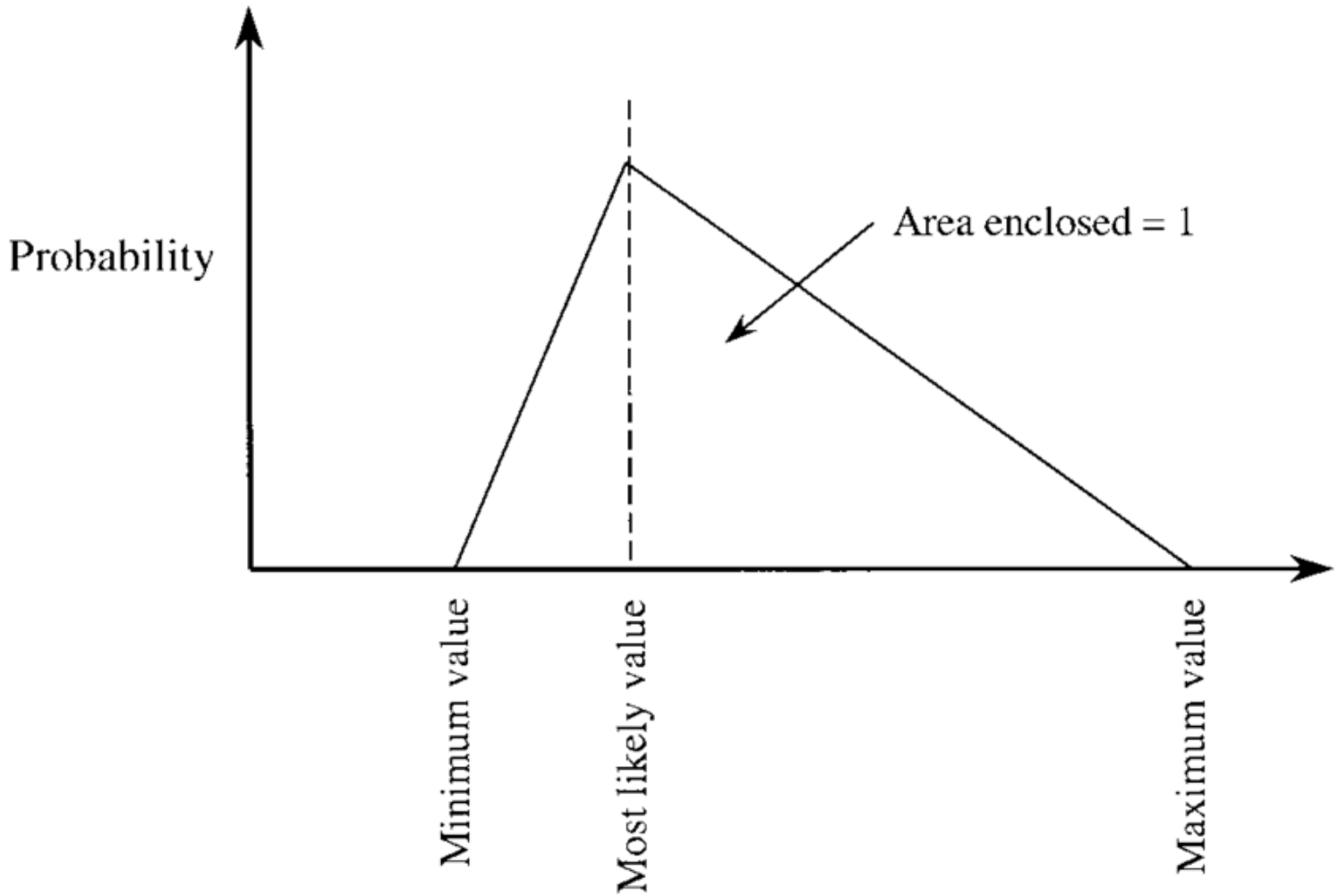
Three Time Estimates

Optimistic Time (t_o)

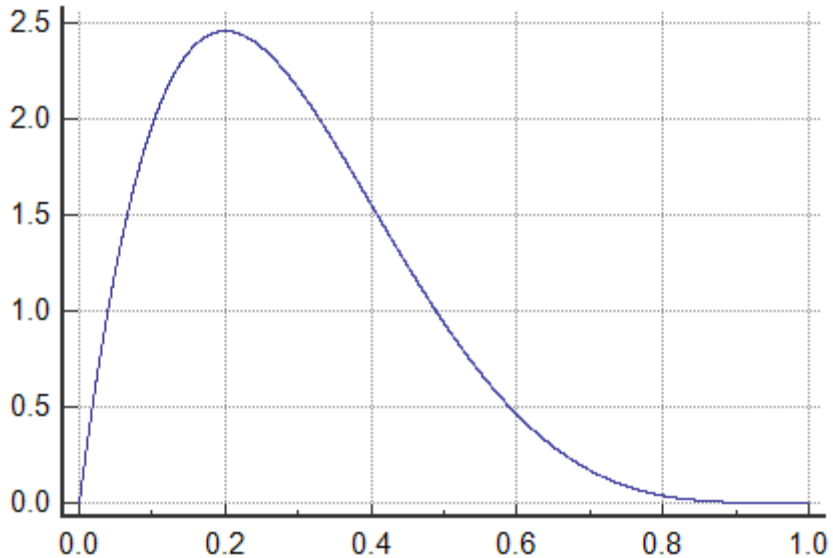
Most Likely Time (t_m)

Pessimistic Time (t_p)

$(t_o) \leq (t_m) \leq (t_p)$


Probability
Area enclosed = 1
Minimum value
Most likely value
Maximum value

Beta Distribution



Find the critical path and project duration for the project.

Activity	Mnemonic	Optimistic time (Days)	Most likely time (Days)	Pessimistic time (Days)	Predecessors (Mnemonic)
Dig Basement	DIG	2	3	4	—
Pour Foundation	FOUND	3	4	5	DIG
Pour Basement Floor	POURB	1	2	3	FOUND
Install Floor Joists	JOISTS	1	3	5	FOUND
Install Walls	WALLS	4	5	6	FOUND
Install Rafters	RAFTERS	2	3	4	WALLS , POURB
Install Flooring	FLOOR	3	4	5	JOISTS
Rough Interior	ROUGH	5	6	7	FLOOR
Install Roof	ROOF	4	6	14	RAFTERS
Finish Interior	FINISH	3	4	11	ROUGH, ROOF
Landscape	SCAPE	1	2	3	POURB, WALLS

Renaming the activity

Activity	Mnemonic	Activity renamed	Optimistic time (Days)	Most likely time (Days)	Pessimistic time (Days)	Predecessors (Mnemonic)	Predecessor
Dig Basement	DIG	A	2	3	4	—	-
Pour Foundation	FOUND	B	3	4	5	DIG	A
Pour Basement Floor	POURB	C	1	2	3	FOUND	B
Install Floor Joists	JOISTS	D	1	3	5	FOUND	B
Install Walls	WALLS	E	4	5	6	FOUND	B
Install Rafters	RAFTERS	F	2	3	4	WALLS , POURB	E, C
Install Flooring	FLOOR	G	3	4	5	JOISTS	D
Rough Interior	ROUGH	H	5	6	7	FLOOR	G
Install Roof	ROOF	I	4	6	14	RAFTERS	F
Finish Interior	FINISH	J	3	4	11	ROUGH, ROOF	H,I
Landscape	SCAPE	K	1	2	3	POURB, WALLS	C, E

Finding the expected time and variance of activities

For Beta distribution

Expected time (Median)

$$\boldsymbol{t_e = \left(\frac{t_o + 4t_m + t_p}{6} \right)}$$

Variance

$$\boldsymbol{\sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2}$$

Expected time and Variance calculation for Activities

Activities	Optimistic time (Days)	Most likely time (Days)	Pessimistic time (Days)	Expected time (Days) $\left(\frac{t_o + 4t_m + t_p}{6}\right)$	Variance $\left(\frac{t_p - t_o}{6}\right)^2$
A	2	3	4	3	$\left(\frac{4-2}{6}\right)^2$ = 1/9
B	3	4	5	$\left(\frac{3+4(4)+5}{6}\right) = 4$	1/9
C	1	2	3	2	1/9
D	1	3	5	3	1/9
E	4	5	6	5	1/9
F	2	3	4	3	1/9
G	3	4	5	4	1/9
H	5	6	7	6	1/9
I	4	6	14	7	$\left(\frac{14-4}{6}\right)^2$ = 25/9
J	3	4	11	5	16/9
K	1	2	3	2	1/9

Replacing the three times by a single expected time estimate

Activity	Mnemonic	Activity renamed	Activity Expected Time (Days)	Predecessors (Mnemonic)	Predecessor
Dig Basement	DIG	A	3	—	-
Pour Foundation	FOUND	B	4	DIG	A
Pour Basement Floor	POURB	C	2	FOUND	B
Install Floor Joists	JOISTS	D	3	FOUND	B
Install Walls	WALLS	E	5	FOUND	B
Install Rafters	RAFTERS	F	3	WALLS , POURB	E, C
Install Flooring	FLOOR	G	4	JOISTS	D
Rough Interior	ROUGH	H	6	FLOOR	G
Install Roof	ROOF	I	7	RAFTERS	F
Finish Interior	FINISH	J	5	ROUGH, ROOF	H,I
Landscape	SCAPE	K	2	POURB, WALLS	C, E

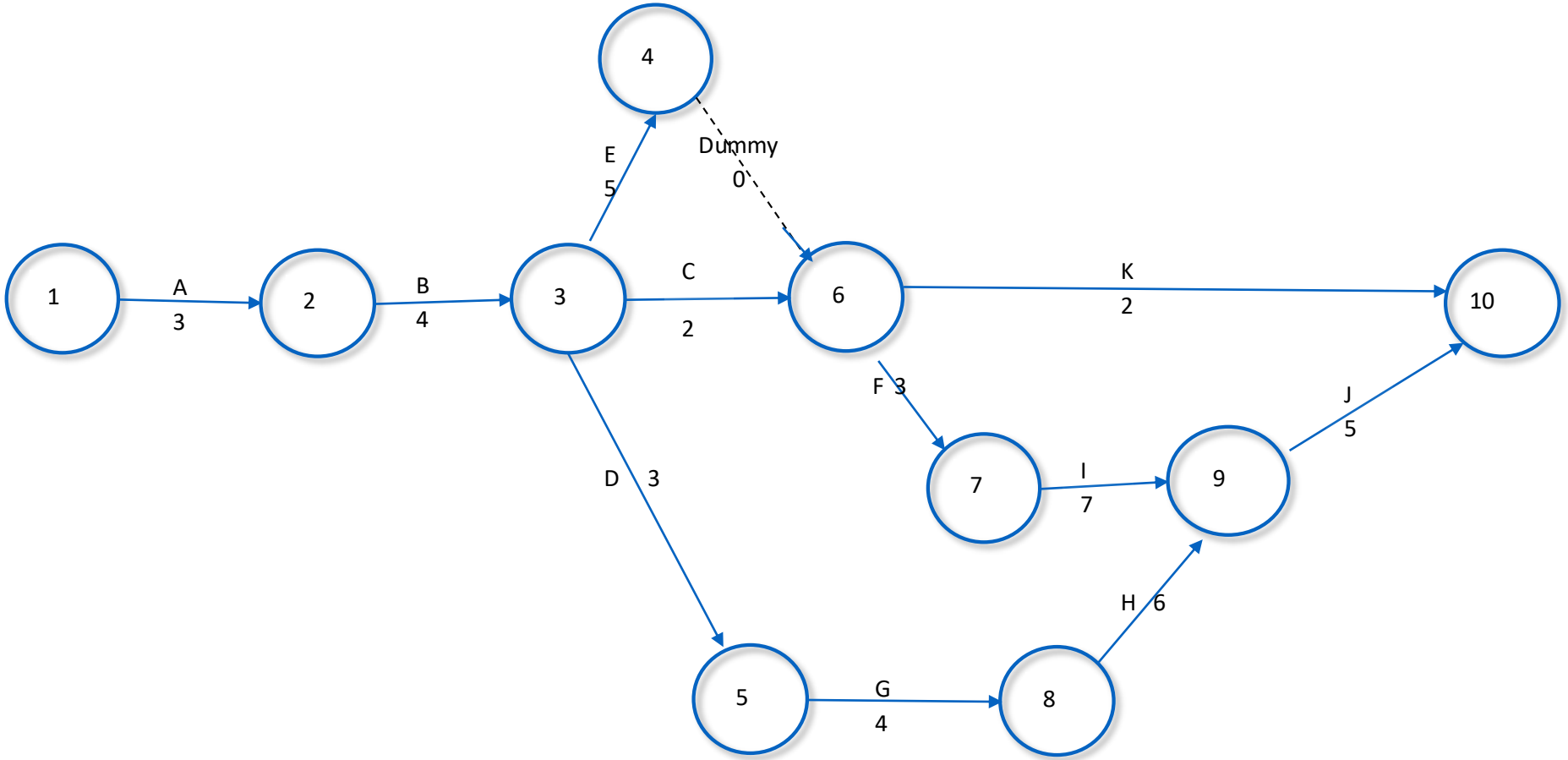
Shorter Version

Activity	Duration (DAYS) Expected time	Predecessor
A	3	-
B	4	A
C	2	B
D	3	B
E	5	B
F	3	E, C
G	4	D
H	6	G
I	7	F
J	5	H,I
K	2	C, E

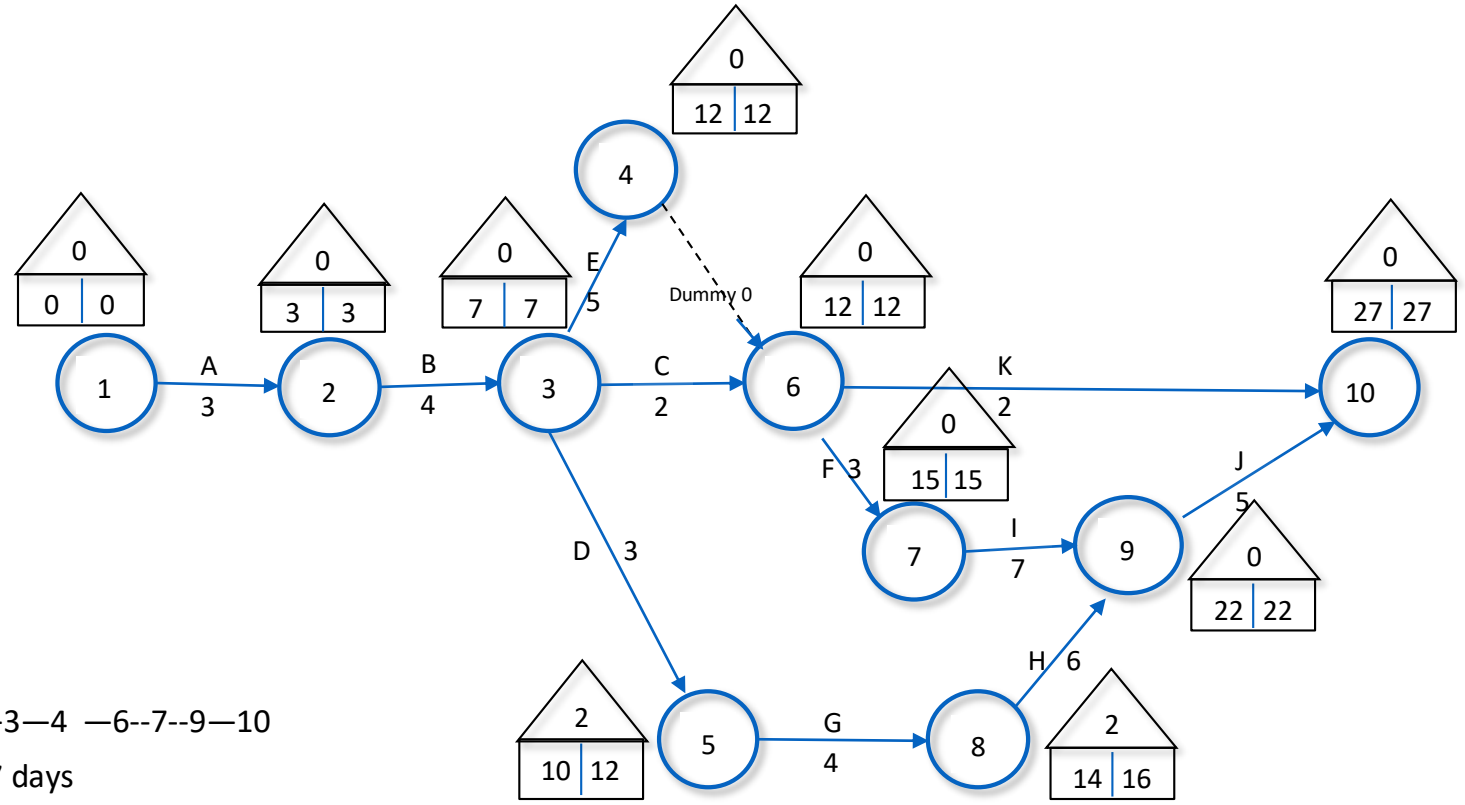
Let us prepare to make a network

Predecessor	Activity	Successor	Duration (Days)
-	A	B	3
A	B	C, D, E	4
B	C	F, K	2
B	D	G	3
B	E	F,K	5
E, C	F	I	3
D	G	H	4
G	H	J	6
F	I	J	7
H,I	J	-	5
C, E	K	-	2

Here is the network



Earliest Start Time (EST) and Latest Finish Time (LFT)

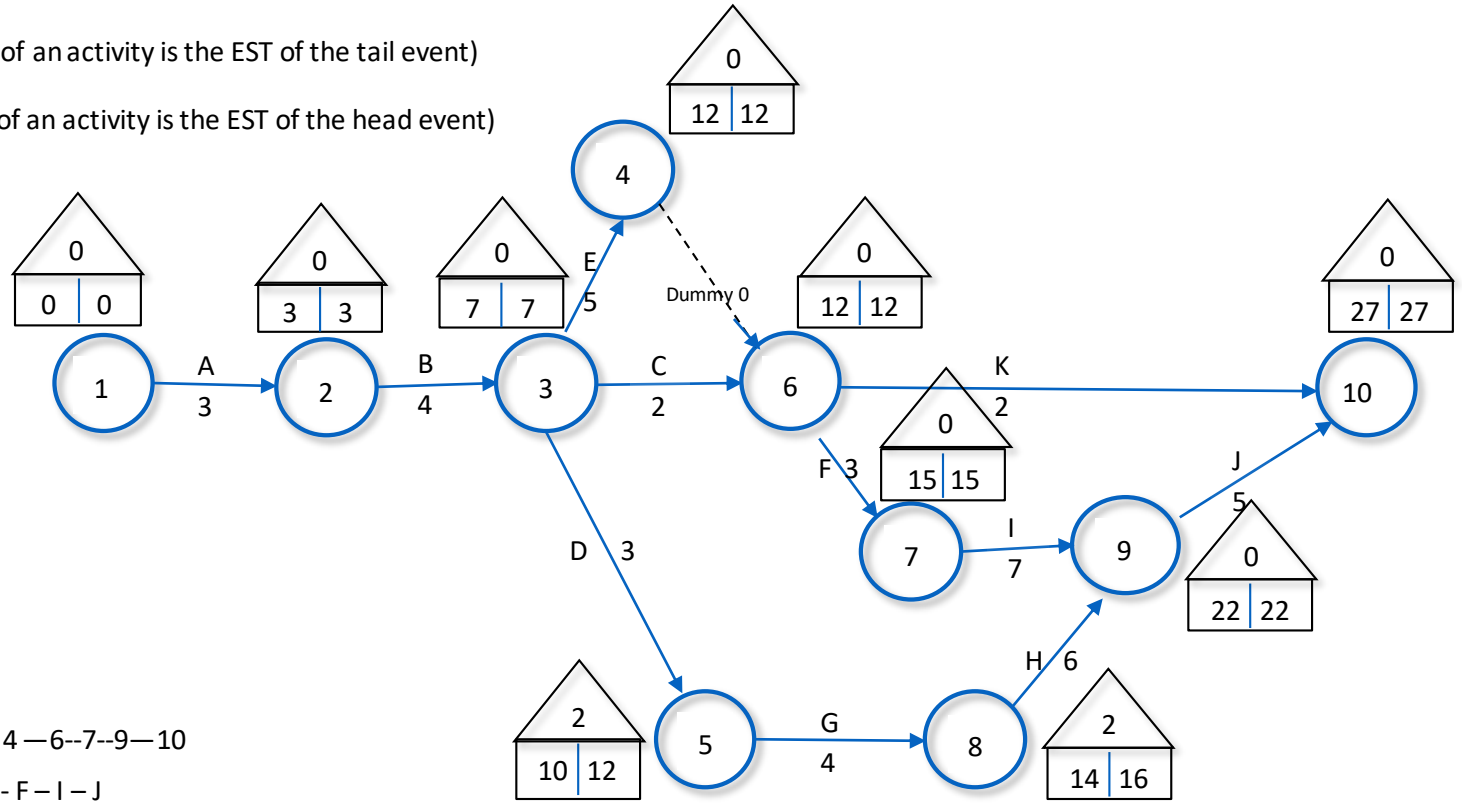


- Critical path is 1—2—3—4 —6--7--9—10
- Project duration is 27 days

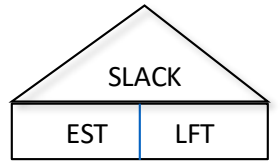
QUIZ: What is the EST and LFT of the activity G?

EST of G is 10 (EST of an activity is the EST of the tail event)

LFT of G is 16 (LFT of an activity is the EST of the head event)



- Critical path is 1—2—3—4—6--7--9—10
- That is A- B - E - DUMMY - F – I – J
- Project duration is 27 days



Activity	Activity (another form)	Duration (Days)	Tail Event Slack	Head Event Slack
A	1-2	3	0	0
B	2-3	4	0	0
C	3-6	2	0	0
D	3-5	3	0	2
E	3-4	5	0	0
Dummy	4-6	0	0	0
F	6-7	3	0	0
G	5-8	4	2	2
H	8-9	6	2	0
I	7-9	7	0	0
J	9-10	5	0	0
K	6-10	2	0	0

EST and LFT calculations of activity

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT (7)	LST (8)	LFT of activity = LFT of head event (9)				
A	1-2	3	0	0	0			3				
B	2-3	4	0	0	3			7				
C	3-6	2	0	0	7			12				
D	3-5	3	0	2	7			12				
E	3-4	5	0	0	7			12				
Dummy	4-6	0	0	0	12			12				
F	6-7	3	0	0	12			15				
G	5-8	4	2	2	10			16				
H	8-9	6	2	0	14			22				
I	7-9	7	0	0	15			22				
J	9-10	5	0	0	22			27				
K	6-10	2	0	0	12			27				

EFT and LST of activities

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)				
A	1-2	3	0	0	0	3	0	3				
B	2-3	4	0	0	3	7	3	7				
C	3-6	2	0	0	7	9	10	12				
D	3-5	3	0	2	7	10	9	12				
E	3-4	5	0	0	7	12	7	12				
Dummy	4-6	0	0	0	12	12	12	12				
F	6-7	3	0	0	12	15	12	15				
G	5-8	4	2	2	10	14	12	16				
H	8-9	6	2	0	14	20	16	22				
I	7-9	7	0	0	15	22	15	22				
J	9-10	5	0	0	22	27	22	27				
K	6-10	2	0	0	12	14	25	27				

Calculation of Total floats

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)	Total Float = EFT-EST = LFT-LST (10) = (7)-(6)			
A	1-2	3	0	0	0	3	0	3	0			
B	2-3	4	0	0	3	7	3	7	0			
C	3-6	2	0	0	7	9	10	12	3			
D	3-5	3	0	2	7	10	9	12	2			
E	3-4	5	0	0	7	12	7	12	0			
Dummy	4-6	0	0	0	12	12	12	12	0			
F	6-7	3	0	0	12	15	12	15	0			
G	5-8	4	2	2	10	14	12	16	2			
H	8-9	6	2	0	14	20	16	22	2			
I	7-9	7	0	0	15	22	15	22	0			
J	9-10	5	0	0	22	27	22	27	0			
K	6-10	2	0	0	12	14	25	27	13			

Calculation of free float

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)	Total Float = EFT-EST = LFT-LST (10) = (7)-(6)	Free Float (11)= (10)-(5)		
A	1-2	3	0	0	0	3	0	3	0	0		
B	2-3	4	0	0	3	7	3	7	0	0		
C	3-6	2	0	0	7	9	10	12	3	3		
D	3-5	3	0	2	7	10	9	12	2	0		
E	3-4	5	0	0	7	12	7	12	0	0		
Dummy	4-6	0	0	0	12	12	12	12	0	0		
F	6-7	3	0	0	12	15	12	15	0	0		
G	5-8	4	2	2	10	14	12	16	2	0		
H	8-9	6	2	0	14	20	16	22	2	2		
I	7-9	7	0	0	15	22	15	22	0	0		
J	9-10	5	0	0	22	27	22	27	0	0		
K	6-10	2	0	0	12	14	25	27	13	13		

Calculation of Independent Float

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)	Total Float = EFT-EST = LFT-LST (10) = (7)-(6)	Free Float = Total Float – Head Event Slack (11)= (10)-(5)	Independent Float (12) = (11)-(4)	
A	1-2	3	0	0	0	3	0	3	0	0	0	
B	2-3	4	0	0	3	7	3	7	0	0	0	
C	3-6	2	0	0	7	9	10	12	3	3	3	
D	3-5	3	0	2	7	10	9	12	2	0	0	
E	3-4	5	0	0	7	12	7	12	0	0	0	
Dummy	4-6	0	0	0	12	12	12	12	0	0	0	
F	6-7	3	0	0	12	15	12	15	0	0	0	
G	5-8	4	2	2	10	14	12	16	2	0	-2	
H	8-9	6	2	0	14	20	16	22	2	2	0	
I	7-9	7	0	0	15	22	15	22	0	0	0	
J	9-10	5	0	0	22	27	22	27	0	0	0	
K	6-10	2	0	0	12	14	25	27	13	13	13	

Calculation of Interfering Float

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)	Total Float = EFT-EST = LFT-LST (10) = (7)-(6)	Free Float = Total Float – Head Event Slack (11)= (10)-(5)	Independent Float (12) = (11)-(4)	Interfering Float (13) = (10) - (11)
A	1-2	3	0	0	0	3	0	3	0	0	0	0
B	2-3	4	0	0	3	7	3	7	0	0	0	0
C	3-6	2	0	0	7	9	10	12	3	3	3	0
D	3-5	3	0	2	7	10	9	12	2	0	0	2
E	3-4	5	0	0	7	12	7	12	0	0	0	0
Dummy	4-6	0	0	0	12	12	12	12	0	0	0	0
F	6-7	3	0	0	12	15	12	15	0	0	0	0
G	5-8	4	2	2	10	14	12	16	2	0	~~-2~~ 0	2
H	8-9	6	2	0	14	20	16	22	2	2	0	0
I	7-9	7	0	0	15	22	15	22	0	0	0	0
J	9-10	5	0	0	22	27	22	27	0	0	0	0
K	6-10	2	0	0	12	14	25	27	13	13	13	0

Calculation of Interfering Float

Activity (1)	Activity (another form) (2)	Duration (Days) (3)	Tail Event Slack (4)	Head Event Slack (5)	EST of activity = EST of tail event (6)	EFT of activity = EST of the activity + duration (7) =(6) +(3)	LST of activity = LFT of the activity - duration (8)	LFT of activity = LFT of head event (9)	Total Float = EFT-EST = LFT-LST (10) = (7)-(6)	Free Float = Total Float – Head Event Slack (11)= (10)-(5)	Independent Float (12) = (11)-(4)	Interfering Float (13) = (10) - (11)
A	1-2	3	0	0	0	3	0	3	0	0	0	0
B	2-3	4	0	0	3	7	3	7	0	0	0	0
C	3-6	2	0	0	7	9	10	12	3	3	3	0
D	3-5	3	0	2	7	10	9	12	2	0	0	2
E	3-4	5	0	0	7	12	7	12	0	0	0	0
Dummy	4-6	0	0	0	12	12	12	12	0	0	0	0
F	6-7	3	0	0	12	15	12	15	0	0	0	0
G	5-8	4	2	2	10	14	12	16	2	0	~~-2~~ 0	2
H	8-9	6	2	0	14	20	16	22	2	2	0	0
I	7-9	7	0	0	15	22	15	22	0	0	0	0
J	9-10	5	0	0	22	27	22	27	0	0	0	0
K	6-10	2	0	0	12	14	25	27	13	13	13	0

Critical path is A- B - E - DUMMY - F – I – J

That is 1—2—3—4 —6--7--9—10

Quiz: In the previous slide, why is the independent float of activity G not -2 but 0?

Answer is in the next slide – last line

What are floats?

- **Total Float (TF):**
  The amount of time that a schedule activity can be delayed or extended from its early start date without delaying the project finish date or violating a schedule constraint.
- Mathematically: Late Finish – Early Finish = Total Float
- **Free Float (FF):**
  The amount of time that a schedule activity can be delayed without delaying the early start date of any successor or violating a schedule constraint.
- Mathematically: Earliest Successors' Early Start – Activity's Early Finish = Free Float
- **Interfering Float (INTF):**
  The amount of time that a schedule activity can be delayed or extended from its early start date without delaying the project finish date, but delaying an activity into interfering float will delay the start of one or more following non-critical activities. If an activity is delayed for the amount of the Free and Interfering Float, its successor activities are critical.
- Mathematically: Total Float – Free Float = Interfering Float
- **Independent Float (INDF):**
  The maximum amount of time an activity can be delayed without delaying the early start of the succeeding activities and without being affected by the allowable delay of any predecessor activity.
- Mathematically: Earliest Successors' Early Start – Earliest Predecessors' Late Finish – Activity's duration = Independent Float
  remark: when the result is a negative value, we set the value to zero.

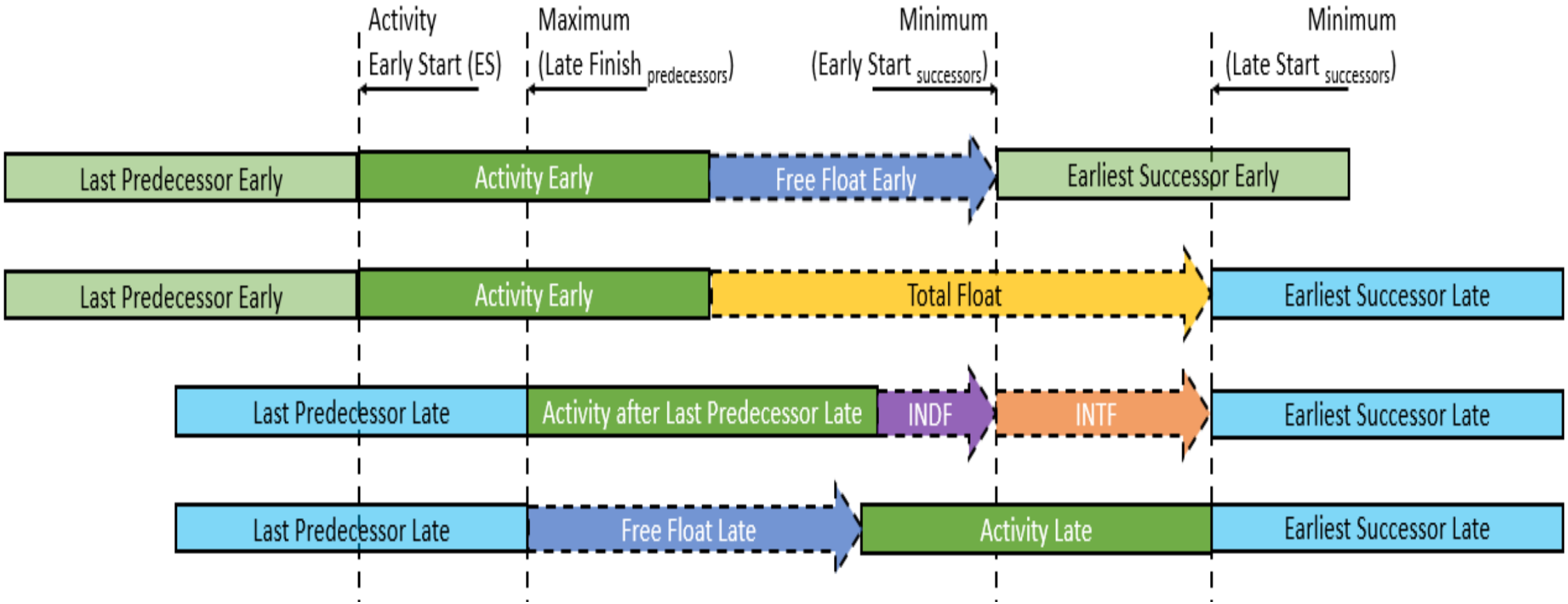
Quiz: If the EST and LFT of the final activity event is same, then which are the activities that will have total float as 0?

Quiz: If the EST and LFT of the final activity event is same, then which are the activities that will have total float as 0?

Answer: Critical activities (that is activities that are on the critical path) and only critical activities will have total float as zero.

Any delay in the critical activity will delay the project.

Schematic representation of TF, FF, INTF, and INDF.



Analysis and ownership

Float	Owner	Consumer
Total Float	owned by the project	Total Float can be consumed either by owner or contractor on a ‘first-come-first-serve’ basis
Free Float	owned by the predecessor activities as much as it’s owned by the activity itself.	Free Float can be consumed by its predecessor activities, if they delay.
Independent Float	genuinely owned by the activity and its owner	consumption of the Independent Float does not affect the surrounding activities’ dates in any possible way.

Calculation of project variance
Critical path A- B - E - DUMMY - F – I – J

Activities	Optimistic time (Days)	Most likely time (Days)	Pessimistic time (Days)	Expected time (Days) $\left(\frac{t_o + 4t_m + t_p}{6}\right)$	Variance $\left(\frac{t_p - t_o}{6}\right)^2$
A	2	3	4	3	$\left(\frac{4-2}{6}\right)^2$ = 1/9
B	3	4	5	$\left(\frac{3+4(4)+5}{6}\right) = 4$	1/9
C	1	2	3	2	1/9
D	1	3	5	3	1/9
E	4	5	6	5	1/9
F	2	3	4	3	1/9
G	3	4	5	4	1/9
H	5	6	7	6	1/9
I	4	6	14	7	$\left(\frac{14-4}{6}\right)^2$ = 25/9
J	3	4	11	5	16/9
K	1	2	3	2	1/9

Calculation of project variance

Project duration is 27 days

Critical path is 1—2—3—4 —6 – 7 - 9—10

That is A- B - E - DUMMY - F – I – J

Project variance = 1/9 + 1/9 + 1/9 + 0 + 1/9 + 25/9 + 16/9 = 45/9

= 5 days

SD = $\sqrt{5}$ days

Probabilities

What is the probability that this project will be completed in 30 days?

Probabilities

What is the probability that the project will be completed in 30 days or earlier?

$$Z = \left(\frac{Completion\ time - duration\ of\ the\ project}{Standard\ seviation}\right)$$ (called as Zee statistics)

$$Z = \left(\frac{30-27}{\sqrt{5}}\right)$$

$Z = 1.342$ (call it as Zee score or Zee value)

P($Z \leq 1.342$) = 0.9099

(For Z-table refer :https://www.statology.org/wp-content/uploads/2018/09/z1.png)

90.99% of completing the project in 30 days or earlier.

Note that, never it will become 100%

Calculation of probabilities

What is the probability of the project to get completed in 27 days?

Probabilities

What is the probability that the project will be completed in 27 days or earlier?

$$Z = \left(\frac{Completion\ time\ -duration\ of\ the\ project}{Standard\ seviation}\right)$$ (called as Zee statistics)

$$Z = \left(\frac{27-27}{\sqrt{5}}\right)$$

$Z = 0$ (call it as Zee score or Zee value)

P($Z \leq 0$) = 0.50

(For Z-table refer :https://www.statology.org/wp-content/uploads/2018/09/z1.png)

50% of completing the project in 30 days

Note that, never it will become 100%

Probabilities

What is the probability that the project will be completed in 25 days or earlier?

$$Z = \left(\frac{Completion\ time - duration\ of\ the\ project}{Standard\ seviation}\right)$$ (called as Zee statistics)

$$Z = \left(\frac{25-27}{\sqrt{5}}\right)$$

$Z = -0.894$ (call it as Zee score or Zee value)

P($Z \leq -0.894$) = 0.1867

(For Z-table refer :https://www.statology.org/wp-content/uploads/2018/09/z1.png)

18.67% of completing the project in 25 days

Note that, never it will become 100%

Crashing of a project

Pre	Task	Normal Time	Crash Time	Normal Cost	Crash Cost	Crash Cost Slope
-	A	2	2	1000	∞	∞
A	B	3	2	5000	6000	1000
A	C	3	2	10000	15000	5000
C	D	4	2	10000	40000	15000
D	E	8	5	17000	50000	4000
B,E	F	6	4	4000	20000	8000
F	G	2	1	5000	10000	5000

The critical path is A-C-D-E-F-G

Thank You for your time