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Abstract

In 1905, a Russian mathematician Andrei A Markov developed a particular class of probabilistic model (also called as stochastic process) where an event depends only on its immediate preceding event rather than other preceding events. These processes are called as Markov chains. In this article, we discuss characteristics of Markov chain, illustration of Markov chain problems with transition probabilities and transition matrix, and finally some real life applications of Markov chains.

Markov Chains

- By Akshay Mohan Joshi

Introduction

In many problems, a sequence of events (or outcomes) are independent of their preceding or succeeding events. In 1905, a Russian mathematician Andrei A Markov developed a particular class of probabilistic model (also called as stochastic process) where an event depends only on its immediate preceding event rather than other preceding events. (Also referred to as one stage dependence) A process (system) of this type is called as a Markov process or chain. A couple of examples,

1. Market share of a product
2. Machines used to manufacture a product

In both these examples, each process/system may be in one of several possible states. These states describe all the possible conditions of the given system. For example,

1. Brand of the product that a customer is using currently is termed as a state
2. The machine condition can be in one of the two possible states: working or not working

Characteristics of a Markov Chain

For a problem to be classified as a Markov chain, following conditions must be satisfied

1. There are finite number of possible states
2. States are both collectively exhaustive and mutually exclusive
3. The transitional probabilities* depend only on the current state of the system
4. The long run probability of being in a particular state will be constant over time
5. The sum of transition probabilities of moving to alternative states in the next time period should add up to one

Markov chains are classified by their order. If the current state depends only on the immediate preceding state, it is said to be a *first order Markov chain*. If it depends upon the last two states, it is called as a *second order Markov chain* and so on.

* *Transitional probability is the conditional probability of the system in moving from one state to another. If transition probabilities don't change from one event to another of the sequence, the Markov chain is said to be *stationary*, otherwise said to be *non-stationary or time dependent*.

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Transition Diagram

Transitional probabilities of the entire system can be represented as elements of a square matrix or by a transition diagram. The matrix helps in predicting the future states of any system under study.

Illustration 1

Let there be 3 brands of soap satisfying the same need and which may be readily substituted for each other. A customer can buy any of the 3 brands of soap at any given point of time.
Hence, there are three possible states corresponding to each brand.
At the time of buying, decision of changing the brand of soap will result in change from one state (brand) to another.
Checklist to confirm if this is a Markov chain problem

- Finite number of states
- States are mutually exclusive and collectively exhaustive
- Change of brand is taken periodically so that changes will occur over a period of time

Let
S_i - State of the system (one of the finite number of possible outcomes)

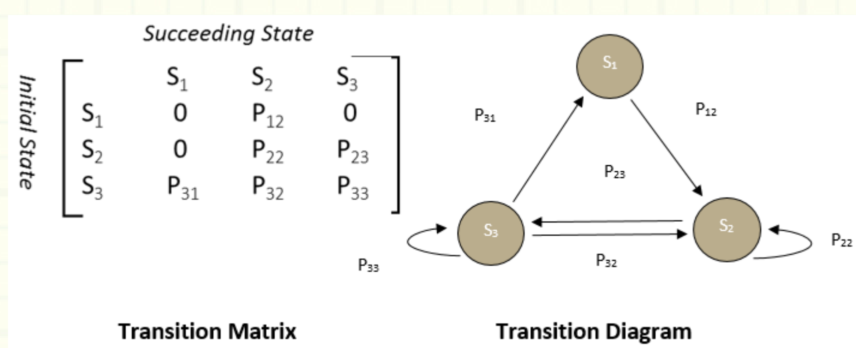
m - Number of states

P_{ij} - Transitional probability of outcome S_j happening, given that event S_i has already happened. That is, probability of being in the state S_j, given that state S_i is the current state of the system

For a Markov chain with 3 states, the matrix of transition probabilities is written as

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \end{array}$$

Transition probabilities can also be represented by the transition diagram as shown where arrows from each state indicate possible transitions and their corresponding probabilities.



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A zero element in the transition matrix indicates that transition is impossible.

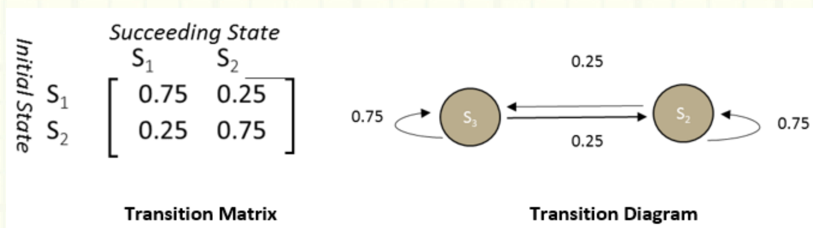
Also, sum of all the probabilities across a row will be equal to one.

i.e.

$$0 + P_{12} + 0 = 1$$
$$0 + P_{22} + P_{23} = 1$$
$$P_{31} + P_{32} + P_{33} = 1$$

Illustration 2

It is sometimes claimed that the best way to predict tomorrow's weather is simply to guess that it is more likely to be same tomorrow as it is today. If we assume that this claim is correct, then it is natural to model the weather as a Markov chain. For simplicity, we assume that there are only two kinds of weather: Rain and Sunshine. If the above predictor is correct 75% of the time, then the weather forms a **Markov Chain** with State Space: $S = s_1, s_2$ with s_1 = "rain" and s_2 = "Sunshine".



Applications of Markov Chain

Markov chains are useful across different fields such as Physics, Chemistry, Economics, Information theory, etc. Following examples illustrate applications of Markov chain in real life,

1. Markovian systems appear extensively in thermodynamics and statistical mechanics, whenever probabilities are used to represent unknown details of the system
2. Crystallization and growth of some epitaxial super lattice oxide materials can be accurately described by Markov chains
3. The LZMA lossless data compression algorithm combines Markov chains with Lempel-Ziv compression to achieve very high compression ratios
4. The Page Rank of a webpage as used by Google is defined by a Markov chain
5. Markov models have also been used to analyze web navigation behavior of users
6. Markov chains are used in finance and economics to model asset prices and market crashes
7. The children's game *Snakes and Ladders* is represented exactly by Markov chains

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Apart from the basic topics covered in this article, learning about multi-period transition probabilities and formulation of transition matrix will be useful to get started on building a Markov chain.

References

- Operations Research: Theory and Applications 5th Edition by J.K. Sharma
- The five greatest applications of Markov chains by Philipp von Hilgers and Amy n. Langville