Game Theory,
Breaking Bad
&
Ice-cream Wars

Prisoners Dilemma

Prisoners Dilemma

Conditions-

- If you both holdout, they get 1 year in jail
- If both confess, they get 2-year jail
- If only one person confesses, that person goes free, and the other person gets 4-year jail for holding out

Game theory is the study of mathematical models of strategic interaction among rational decision-makers.

Applications- Psychology, Evolutionary Biology, War Strategy, Politics, Economics, Business, Policy Making

	Hold Out	Confess
Hold Out	-1	-4
Confess	0*	-2*

Agenda
- Nash Equilibrium
- Decision Theory vs Game Theory
- Approach
- Mix Strategy algorithm
- Infinite Games

Game theory is the study of mathematical models of strategic interaction among rational decision-makers.

Applications- Psychology, Evolutionary Biology, War Strategy, Politics, Economics, Business, Policy Making

Strict Dominance N.E-
Confess > Holding out

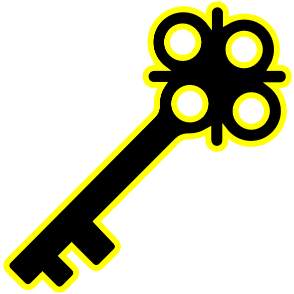
	Hold Out	Confess
Hold Out	**-1 , -1**	**-4 , 0^***
Confess	**0^* , -4**	**-2^* , -2^***

Multiple N.E with Various dimensions in real life

Nash Equilibrium (John Nash)- Optimal outcome of a game where there is no incentive to deviate from the initial strategy(No Regrets Strategy)

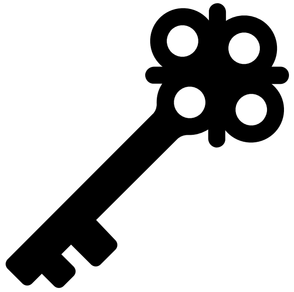
  

Monte Hall problem



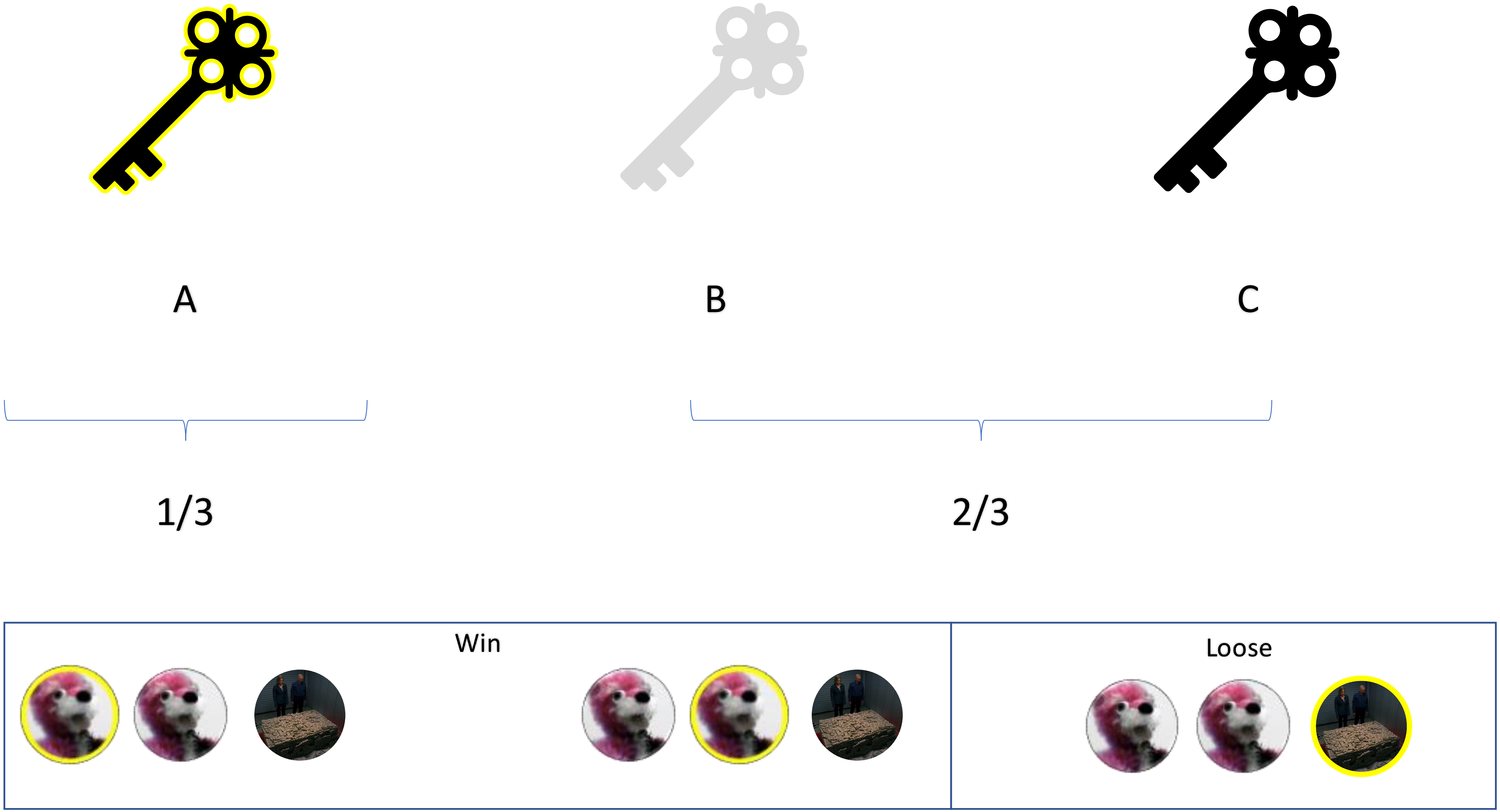
A

B



C

Will you switch?

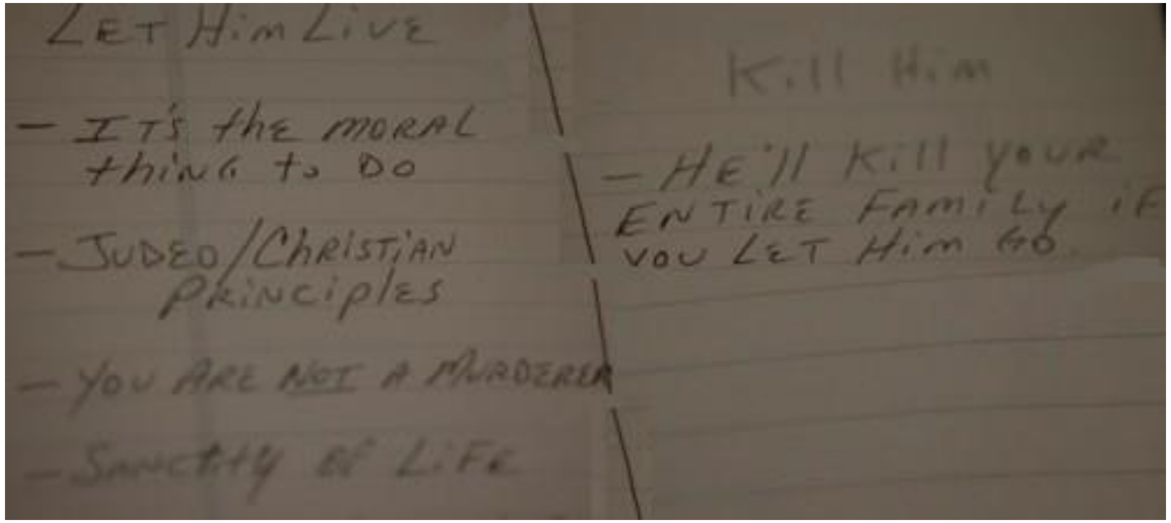

A
B
C
1/3
2/3
Win
Loose

Decision Theory vs Game Theory

Decision Theory (Static)	Game Theory (Dynamic)
Make best possible decision from given data	Your actions(or thinking) affect what others will do. Anticipate reaction from others, make best decision
Ex- “Will stay up late for faster internet”	“Everyone will stay up late for faster internet and slow it down. Therefore, wake up early instead”
‘Sell drugs to make money’	‘Will make enemies while selling drugs. Think strategically at every step to stay ahead and keep his family safe’

Scene 1:

Krazy 8 held hostage while trying to kill Walt and Jesse. Walt was deciding what to do



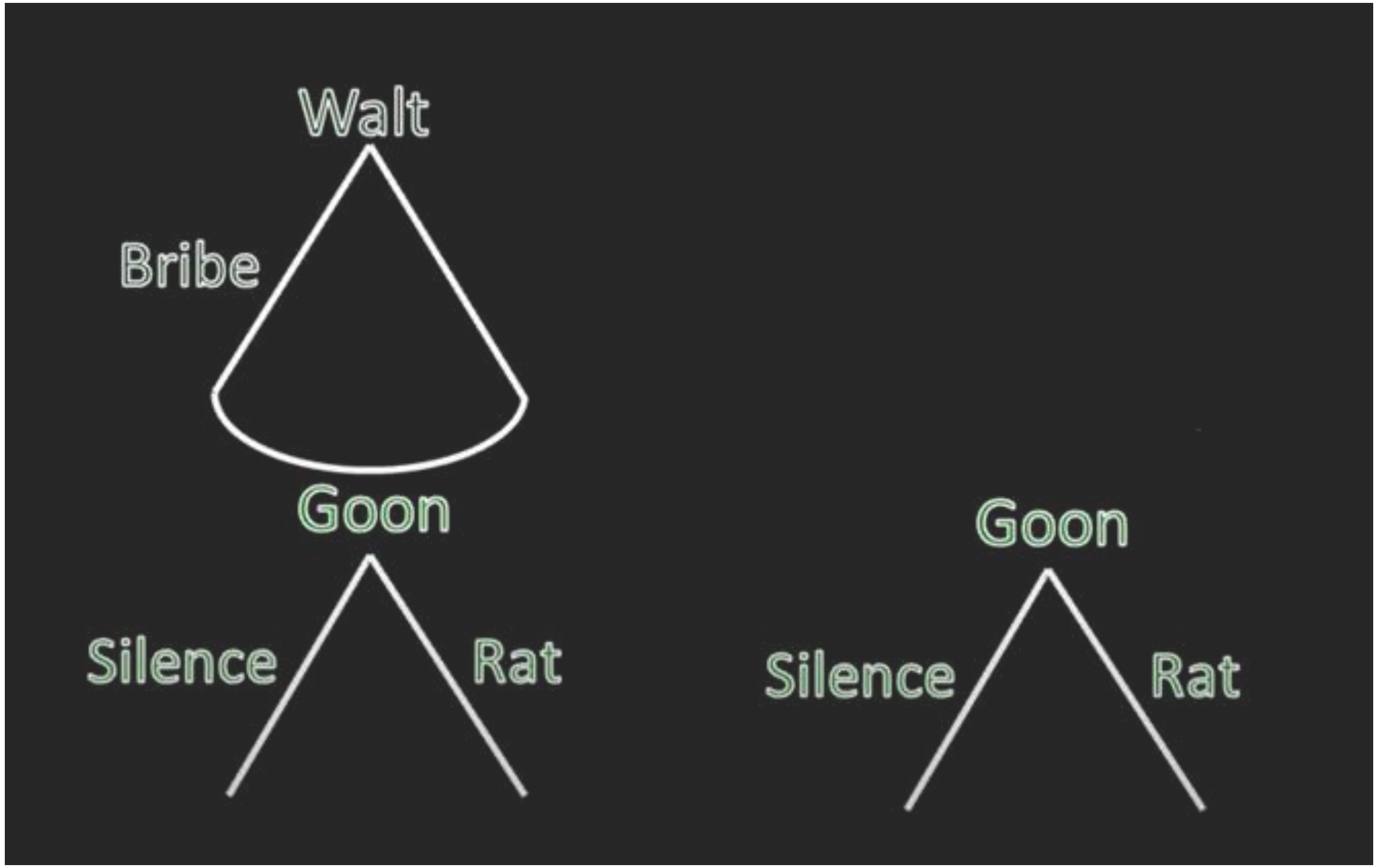
Decision Theory (Static)	Game Theory (Dynamic)
Moral, Guilt, Principles	Repercussions if Krazy8 lives
Limited data, trying to make best possible decision	Asses how would he would react on his decision

Scene 2:

Jesse plans to kill Tuco. Walt asks him details of the plan

Decision Theory (Static)	Game Theory (Dynamic)
Get gun. Shoot Tuco	How? How will his people react? What if you don't shoot him right? Have you ever used gun before?
Makes plan with data	Anticipating Tuco and his people's reaction.
Result- Very difficult	Will need more sophisticated attack

Scene 3:
Mike pays money to prisoners in custody. Hank figures out that the operation is still on-going



Blowfish, Mutually Assured Destruction

Evil Account

Performance(out of 100)= `random.randrange(0, 80) + X`

You have opportunity to get extra points(X)

Conditions-

- Select whether you want 5 or 20
- If more than 20% people pick 20, no one gets any points
- Selections would be completely anonymous to rest of the team

Strategy-

If we cooperate, everyone can get at least 5?

Other’s picking 20

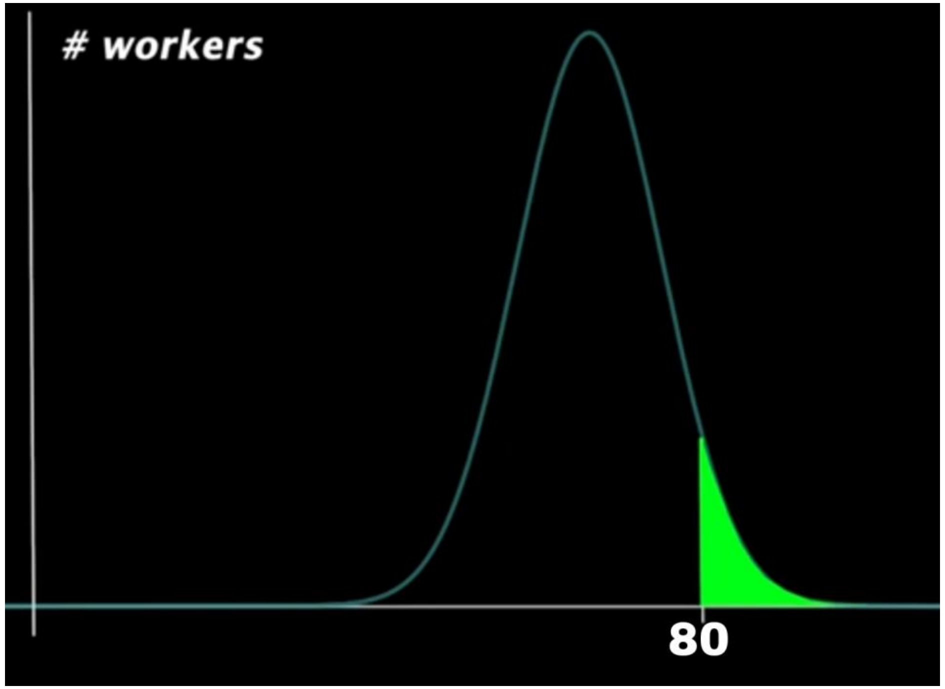
Your Pick		>20%	<20%	=20%
	5 Points	0	5	5*
	20 points	0*	20*	0

		Other's picking 20	
	>20%	**<20%**	**=20%**
Your Pick **5 Points**	0	5	5*
Your Pick **20 points**	0*	20*	0

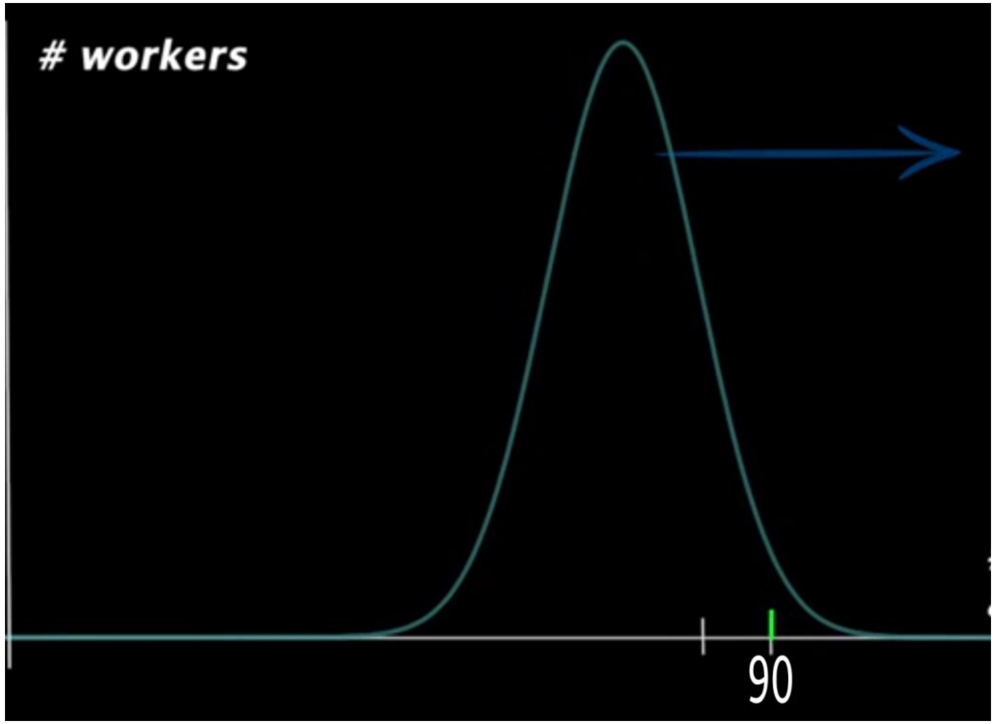
5-5	0 extra
20-5	15 extra

Bonus Motivation

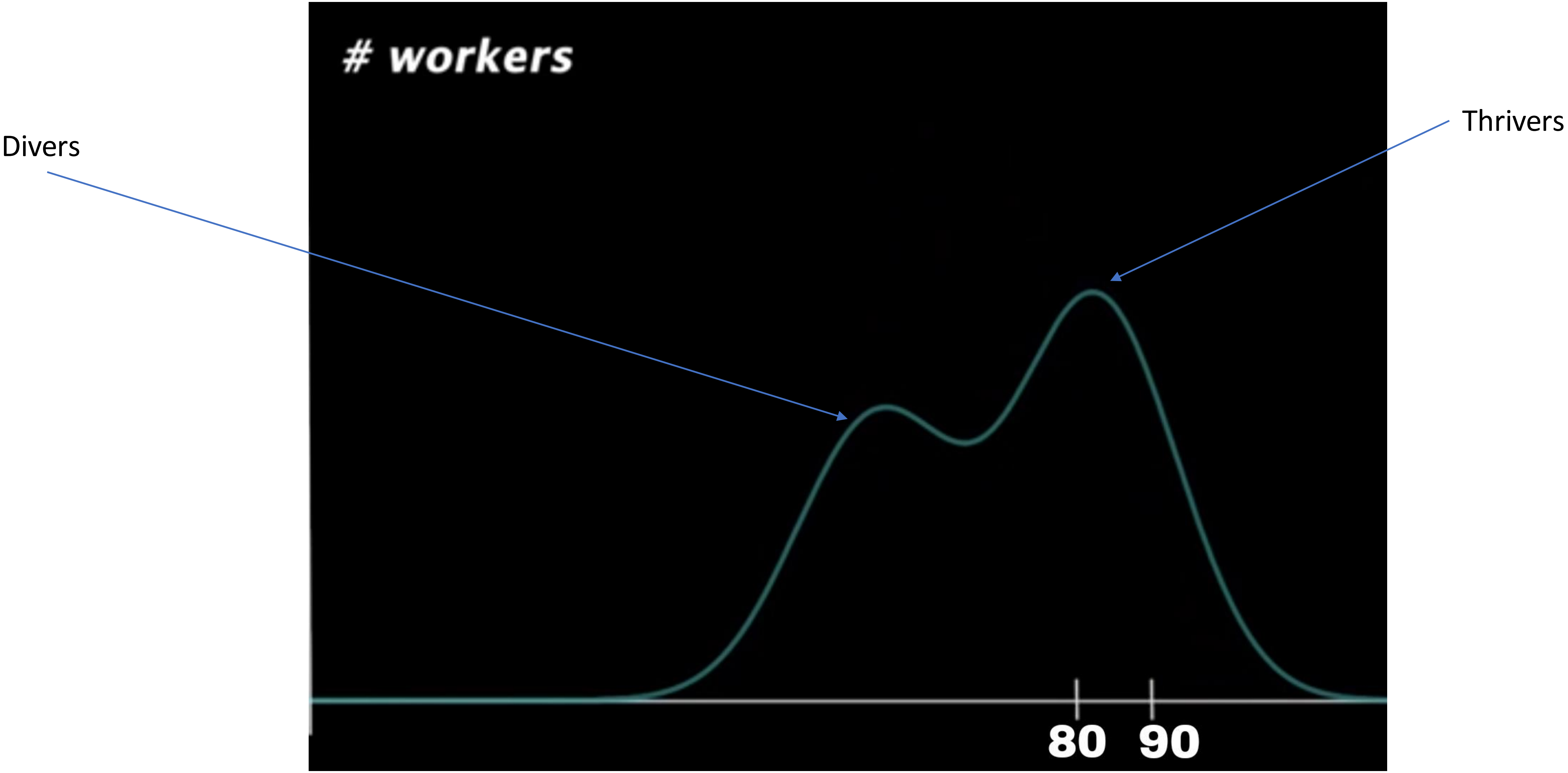
Conditions: Employees reaching a target P score of 80 would receive 2L bonus



Modification: Employees reaching a target P score of 90 would receive 2L bonus

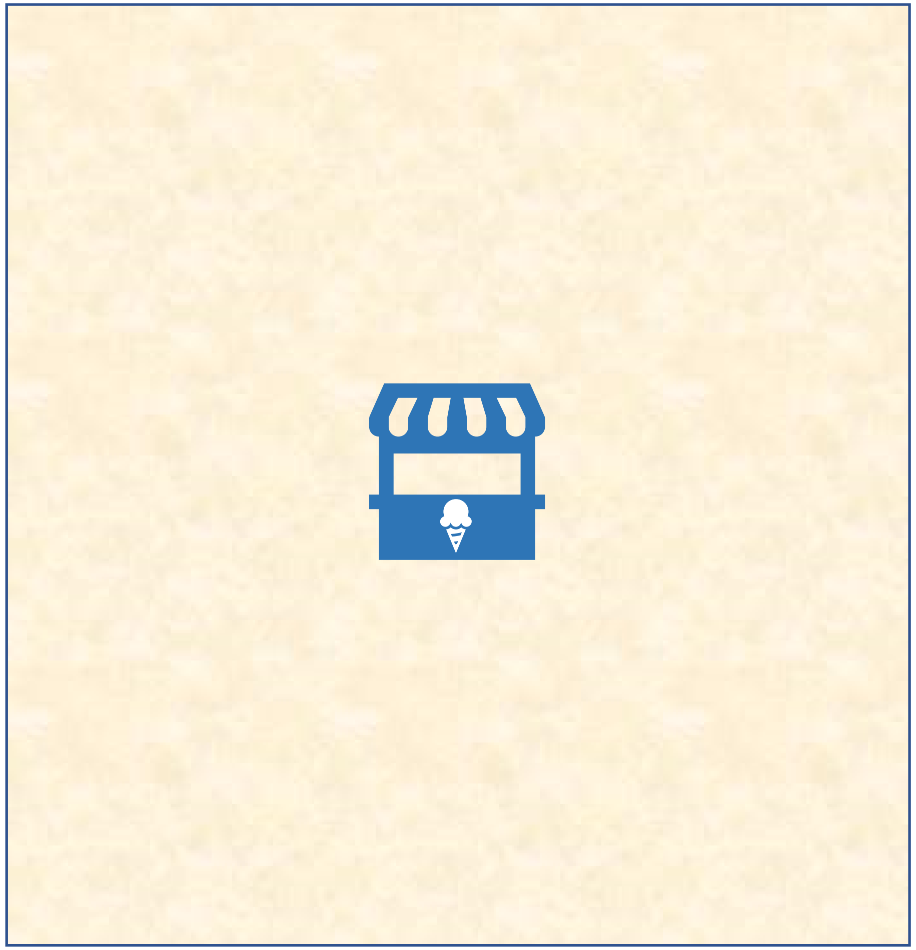


?

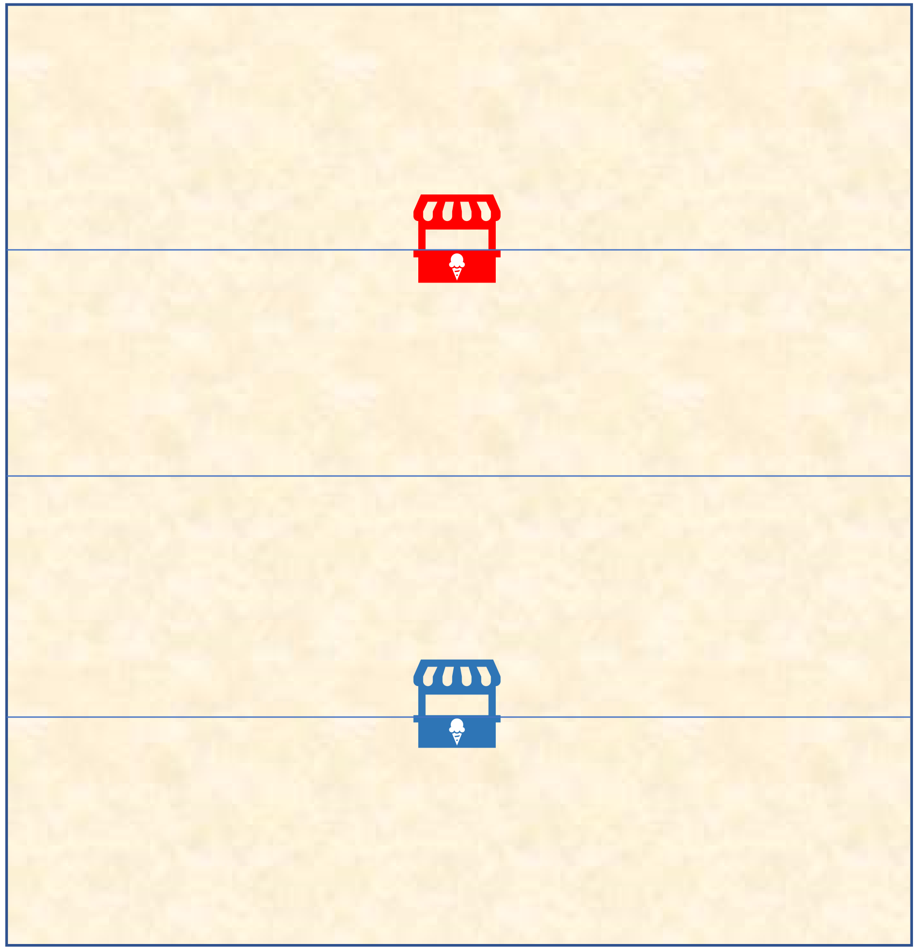


What will happen if there are 3 bonus bands?

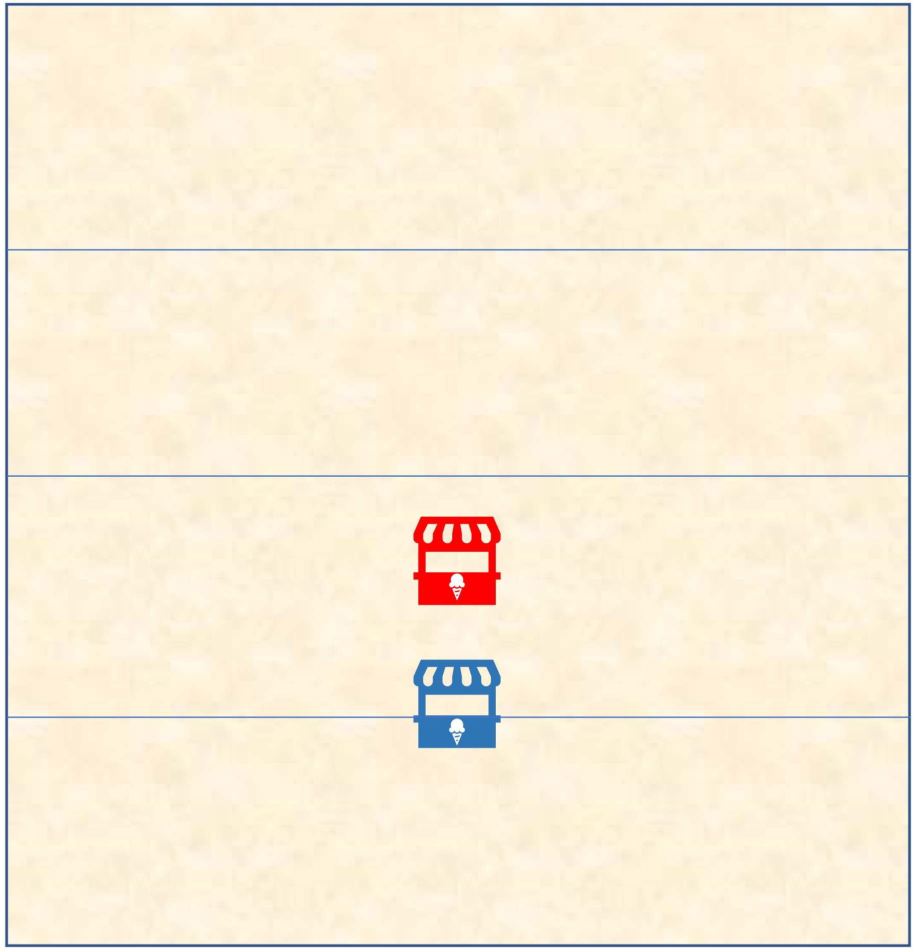
Ice-cream Battle



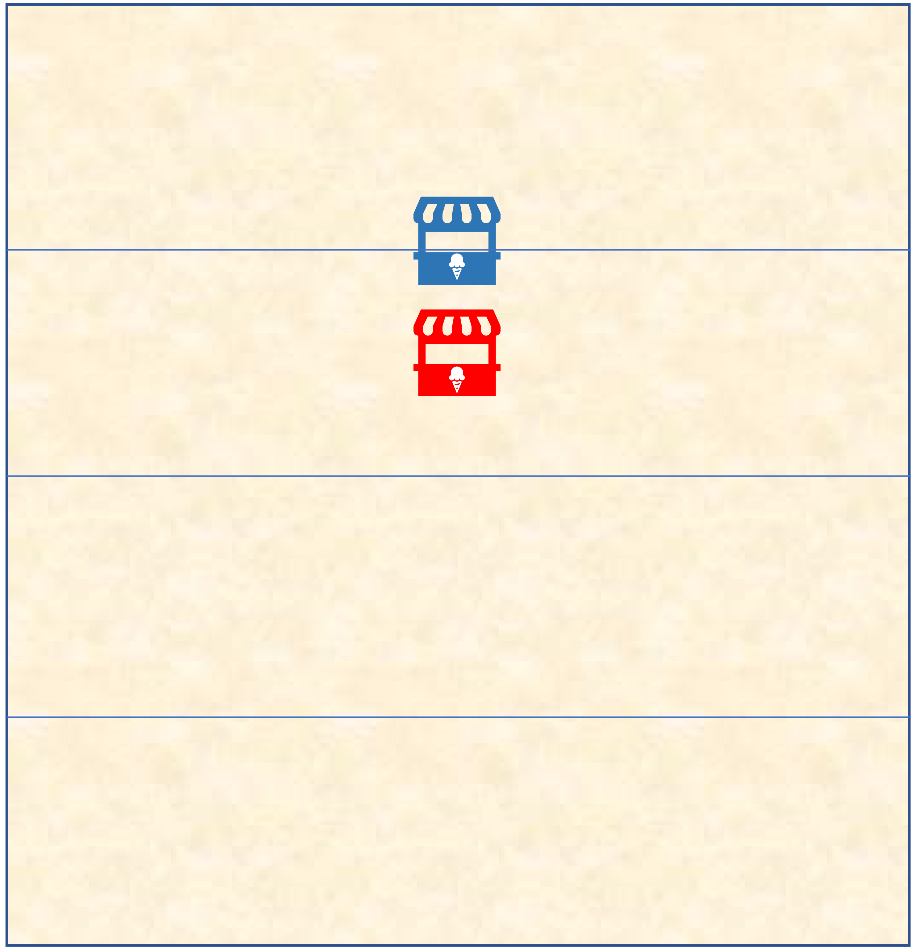
Socially Optimal Solution



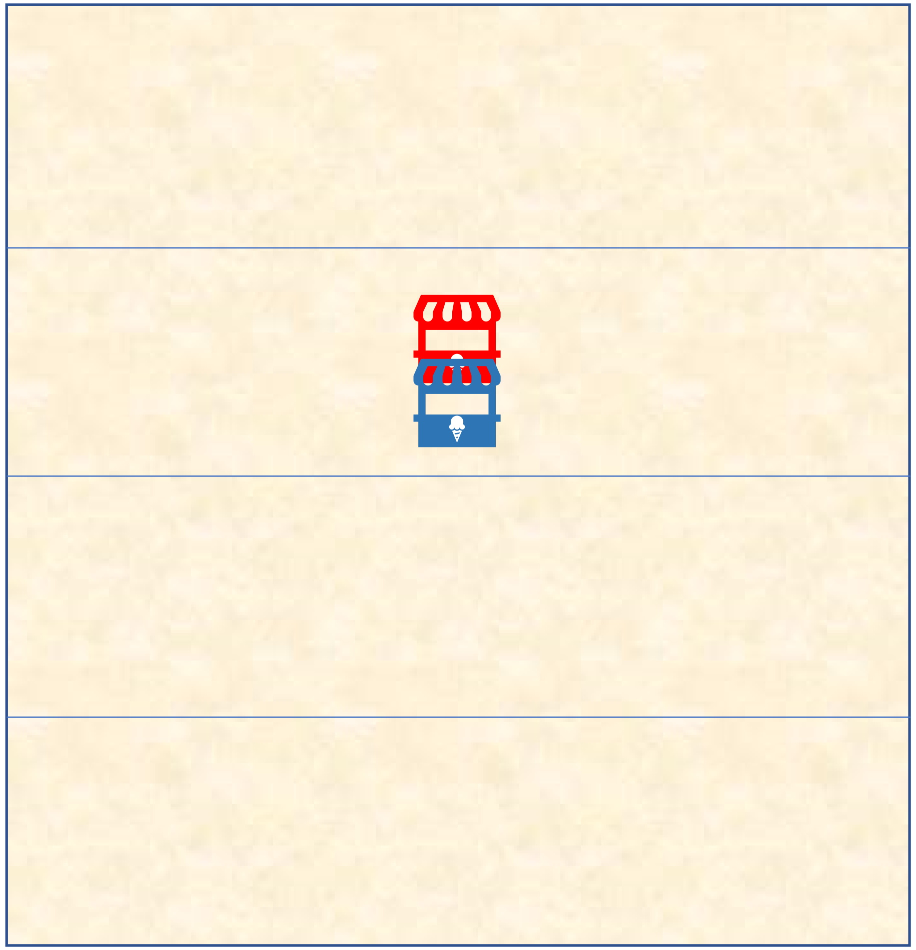
Day-1

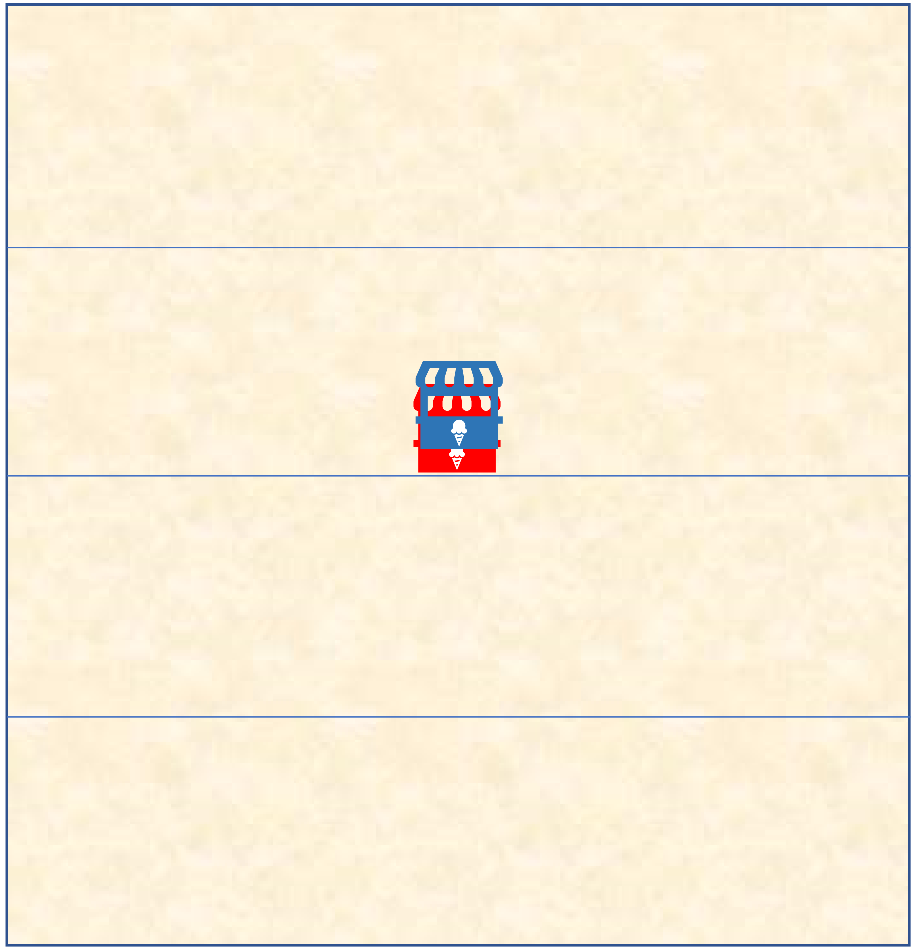


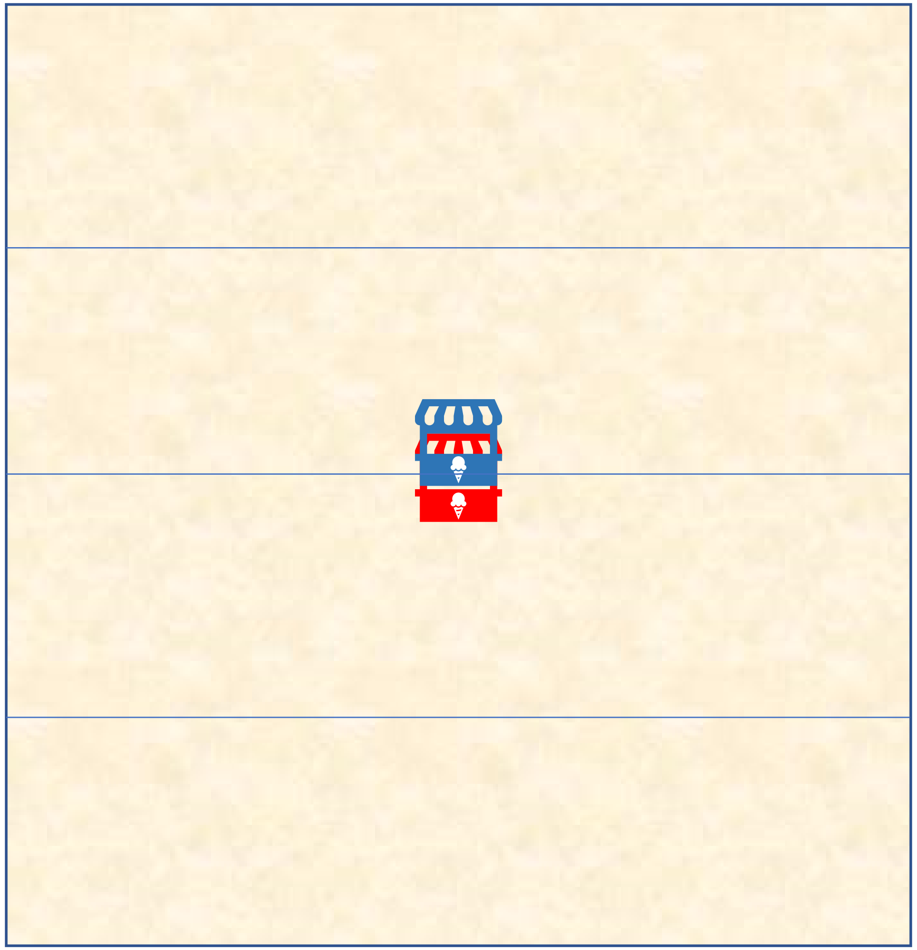
Day-2



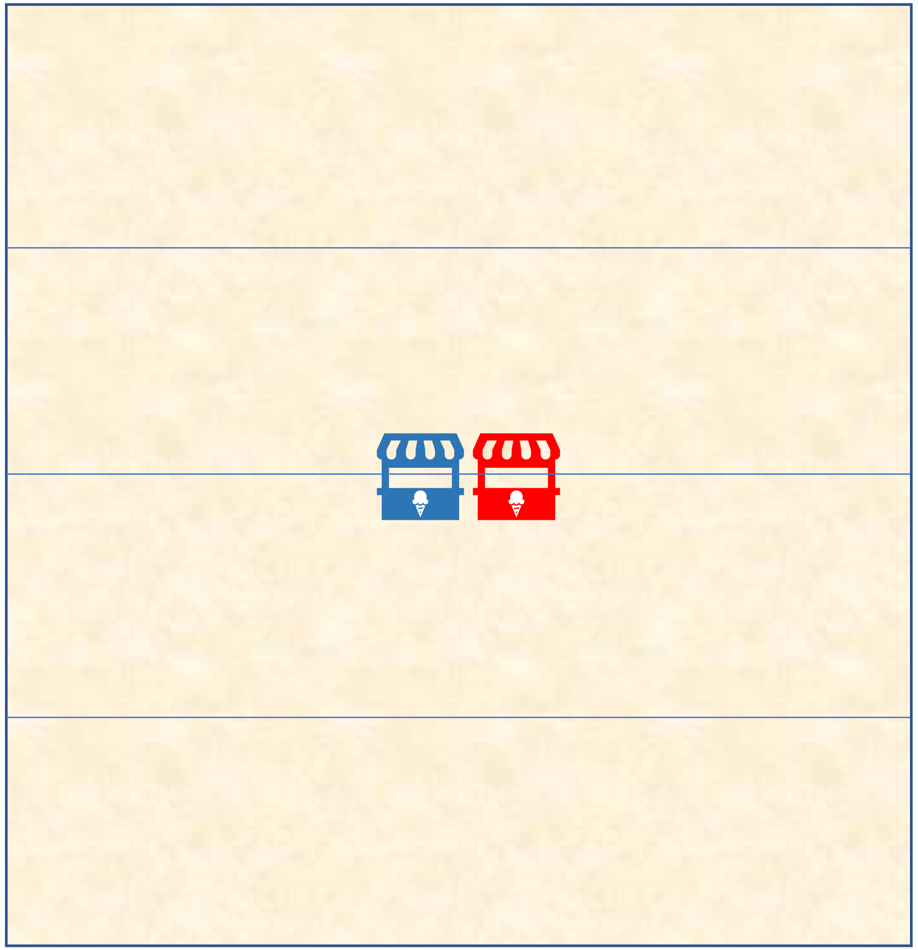
Day-3







Nash Equilibrium

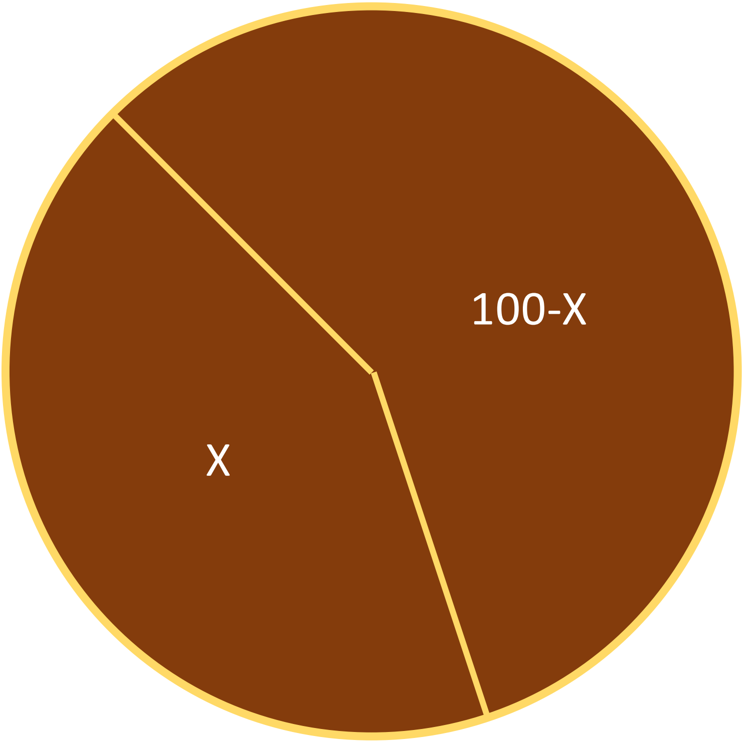


Clustering of stores, companies, stalls etc

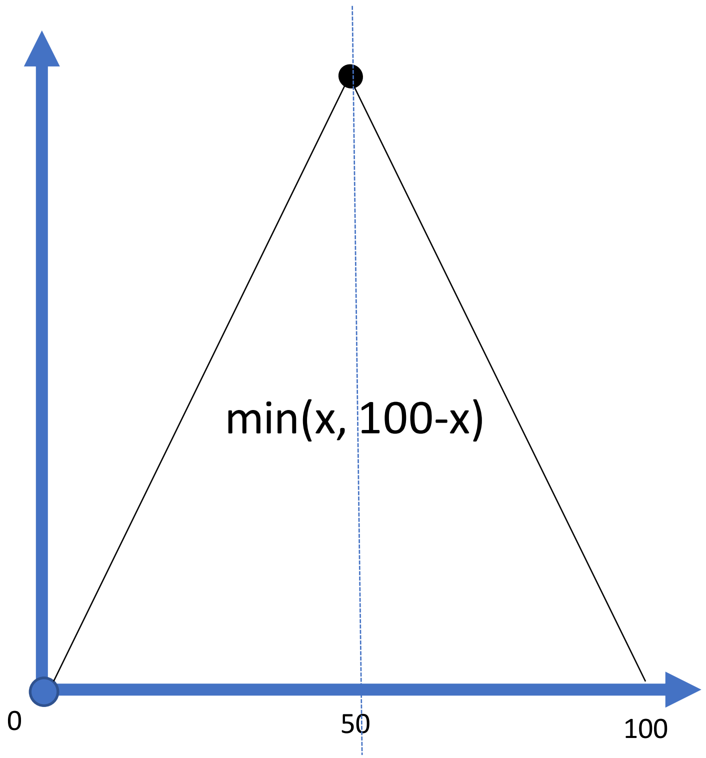
Splitting Cake

Conditions: I cut, you choose

B : chooses max(x, 100-x)

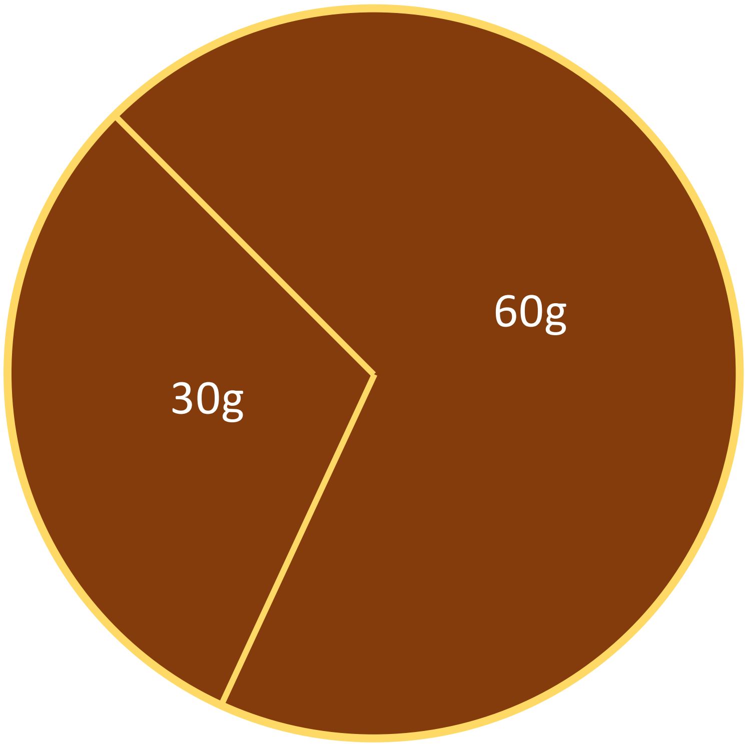


A: Gets min(x, 100-x)



Splitting Cake – 3 people

Conditions: Person A cuts a slice, Person B can reduce it further, Person C can reduce it too. Last person to cut gets the slice



Last Diminisher method

If anyone makes a portion lesser than 30g, they end up with less than even split

If anyone makes a portion bigger than 30g, the last person will get more than an even split

Matching Pennies

Conditions-

- You and your friend simultaneously reveal a coin
- If both show head or both show tails, you get Rs. 100 from him
- If one shows heads and other shows tails, you pay him Rs. 100

Player 2

Player 1		Heads	Tails
	Heads	(100, -100)	(-100, 100)
	Tails	(-100, 100)	(100, -100)

Matching Pennies

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Player 1		Heads	Tails
	Heads	(100, -100)	(-100, 100)
	Tails	(-100, 100)	(100, -100)

Zero Sum Game

Mixed Strategy Nash Equilibrium- (0.5,0.5)

Matching Pennies- Mixed Strategy Algorithm

Player 2

	Left	Right
Player 1 — Up	(300, -300)	(-200, 200)
Player 1 — Down	(-100, 100)	(0, 0)

Matching Pennies- Mixed Strategy Algorithm

Player 2

		Left	Right
Player 1	Up	(300, -300)	(-200, 200)
	Down	(-100, 100)	(0, 0)

Player 1's Mixed Strategy

EU$_L$= EU$_R$

EU$_L$ $= f(\sigma_U)$ σ_U = Probability that P1 plays Up

EU$_R$ $= f(\sigma_U)$

Player 2's Mixed Strategy

EU$_U$= EU$_D$

EU$_U$ $= f(\sigma_L)$ σ_L = Probability that P2 plays Left

EU$_D$ $= f(\sigma_L)$

Player 1's Mixed Strategy

EU$_L$= EU$_R$

EU$_L$ $= \sigma_U(-3) + (1 - \sigma_U)(1)$

EU$_R$ $= \sigma_U(2) + (1 - \sigma_U)(0)$

$\sigma_U = 1/6$

If P1 plays Up 1/6 times and down 5/6 times, P2 is indifferent to P1s moves

Player 2's Mixed Strategy

EU$_U$= EU$_D$

EU$_U$ $= \sigma_L(3) + (1 - \sigma_U)(-2)$

EU$_D$ $= \sigma_L(-1) + (1 - \sigma_U)(0)$

$\sigma_L = 1/3$

If P2 plays left 1/3 times and right 2/3 times, P1 is indifferent to P2s moves

< **(** $\sigma_U = 1/6, \sigma_D = 5/6$ **)** , **(** $\sigma_L = 1/3, \sigma_R = 2/3$ **)** >

Infinite Prisoners Dilemma

Conditions- Goes on for infinite duration

Grim Strategy- Cooperate then defect forever

Grim Trigger- Point after which they would defect forever

δ (Discount Factor) = Measure of how much the Players care about future

Player 2

		Cooperate	Defect
Player 1	Cooperate	**3 , 3**	**1 , 4**
	Defect	**4 , 1**	**2 , 2**

Equilibrium payoff= $3 + 3\delta^1 + 3\delta^2 + 3\delta^3$ ....

Defection payoff= $4 + 2\delta^1 + 2\delta^2 + 2\delta^3$ ....

Infinite Prisoners Dilemma

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Player 2

		Cooperate	Defect
Player 1	Cooperate	**3 , 3**	**1 , 4**
	Defect	**4 , 1**	**2 , 2**

Equilibrium payoff= $3 + 3\delta^1 + 3\delta^2 + 3\delta^3 \ldots$

Defection payoff= $4 + 2\delta^1 + 2\delta^2 + 2\delta^3 \ldots$

Cooperation if

Eqm. payoff>= Def payoff

$\delta >= 1/2$

Applications: Businesses, Trade Wars, Military etc.

Takeaways

- Strategic decision making when everyone around is smart(Prisoners Dilemma)
- How to stay least affected from other people's strategy(Payoff matrix, Mix strategies)
- Realising when to switch strategies (Grim triggers)