



Mu Sigma

# Thursday Learning Hour

*Math Series: Introduction to Linear Algebra- Session 1*

Do The Math

Chicago, IL

Bangalore, India

[www.mu-sigma.com](http://www.mu-sigma.com)

8<sup>th</sup> September 2022

# Basic geometric transformation



Turn



Flip



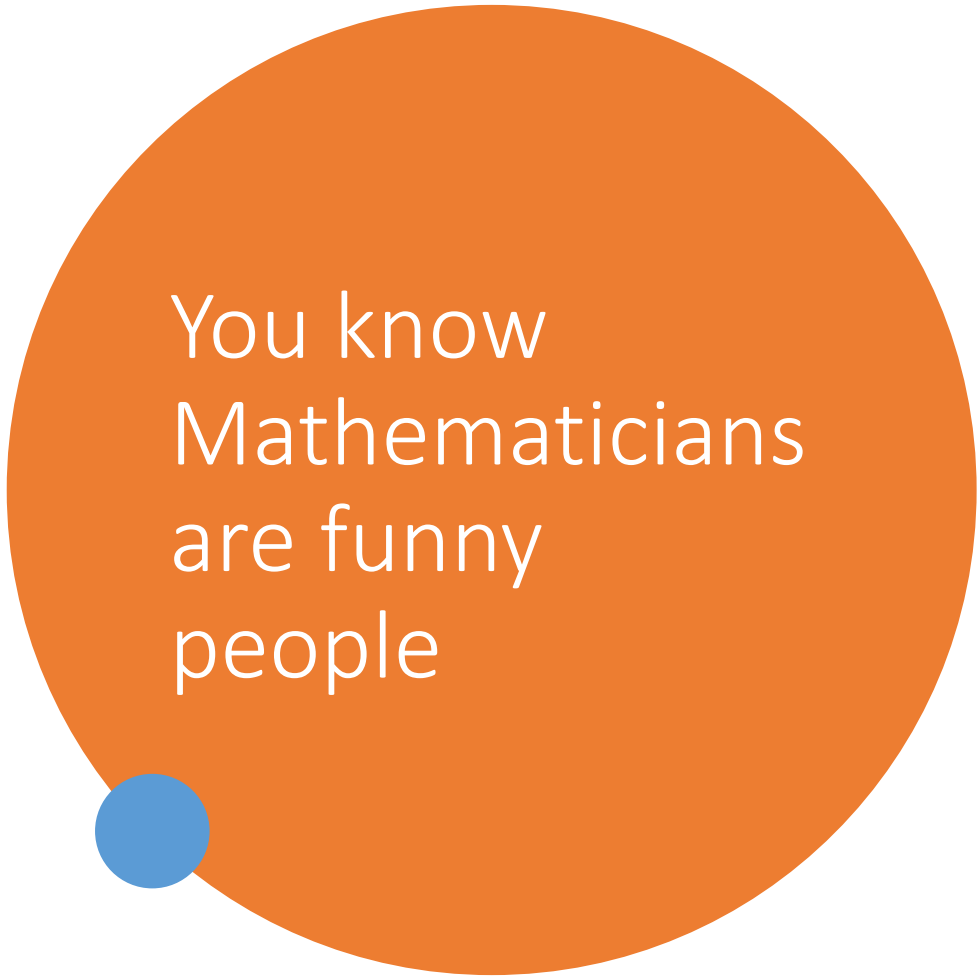
Slide



Resize



Shear



You know  
Mathematicians  
are funny  
people

Turn

Flip

Slide

Resize

Shear

Rotate

Reflection

Translation

Dilation

Skew



Quiz - name the geometric transformation



Quiz - name the geometric transformation

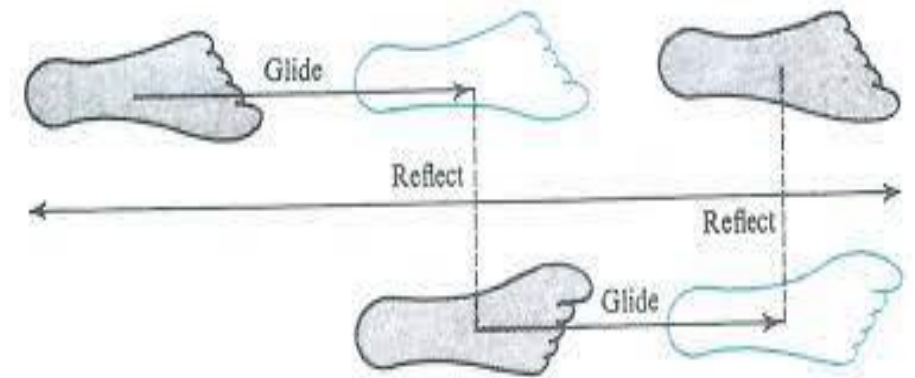
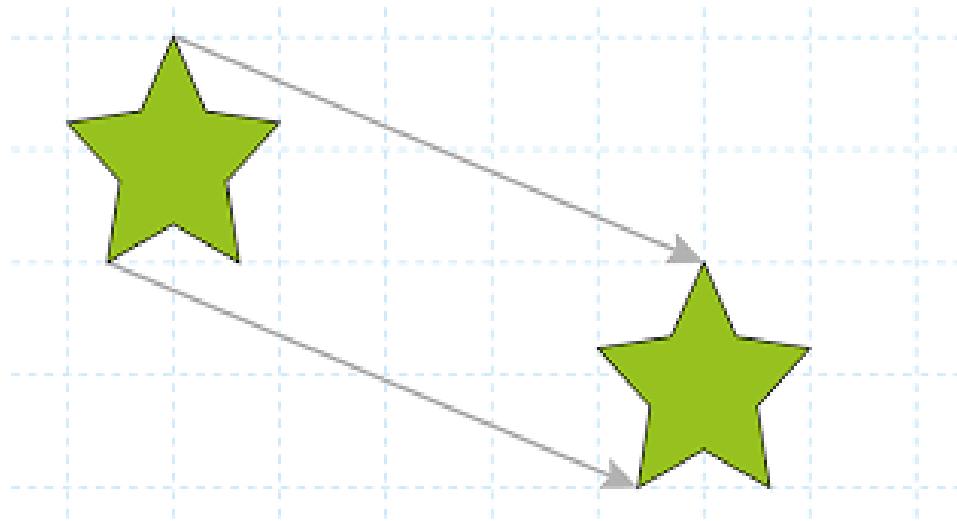




Quiz - name the geometric transformation



What is this?

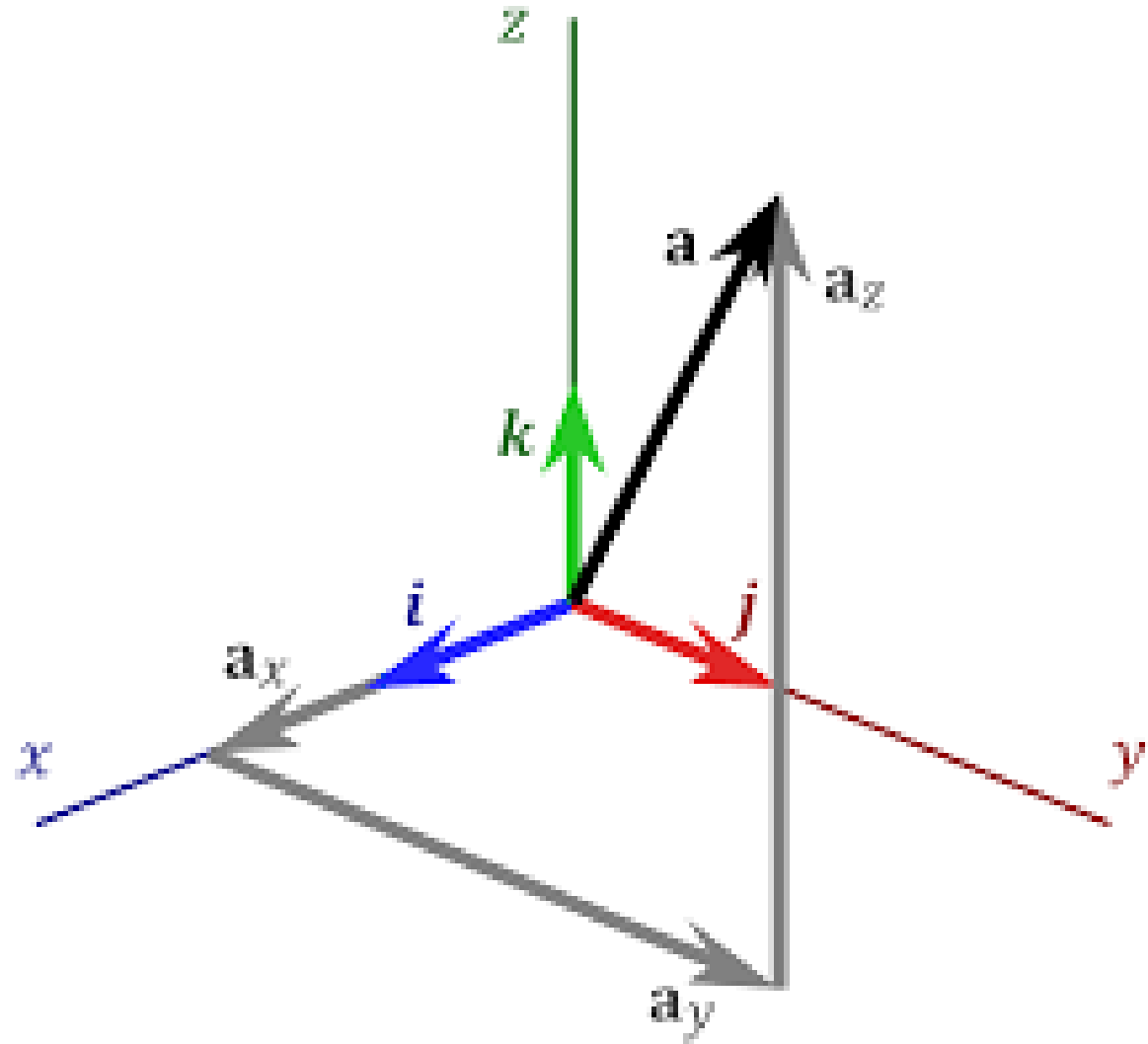


Basic transformations can be represented in a matrix form

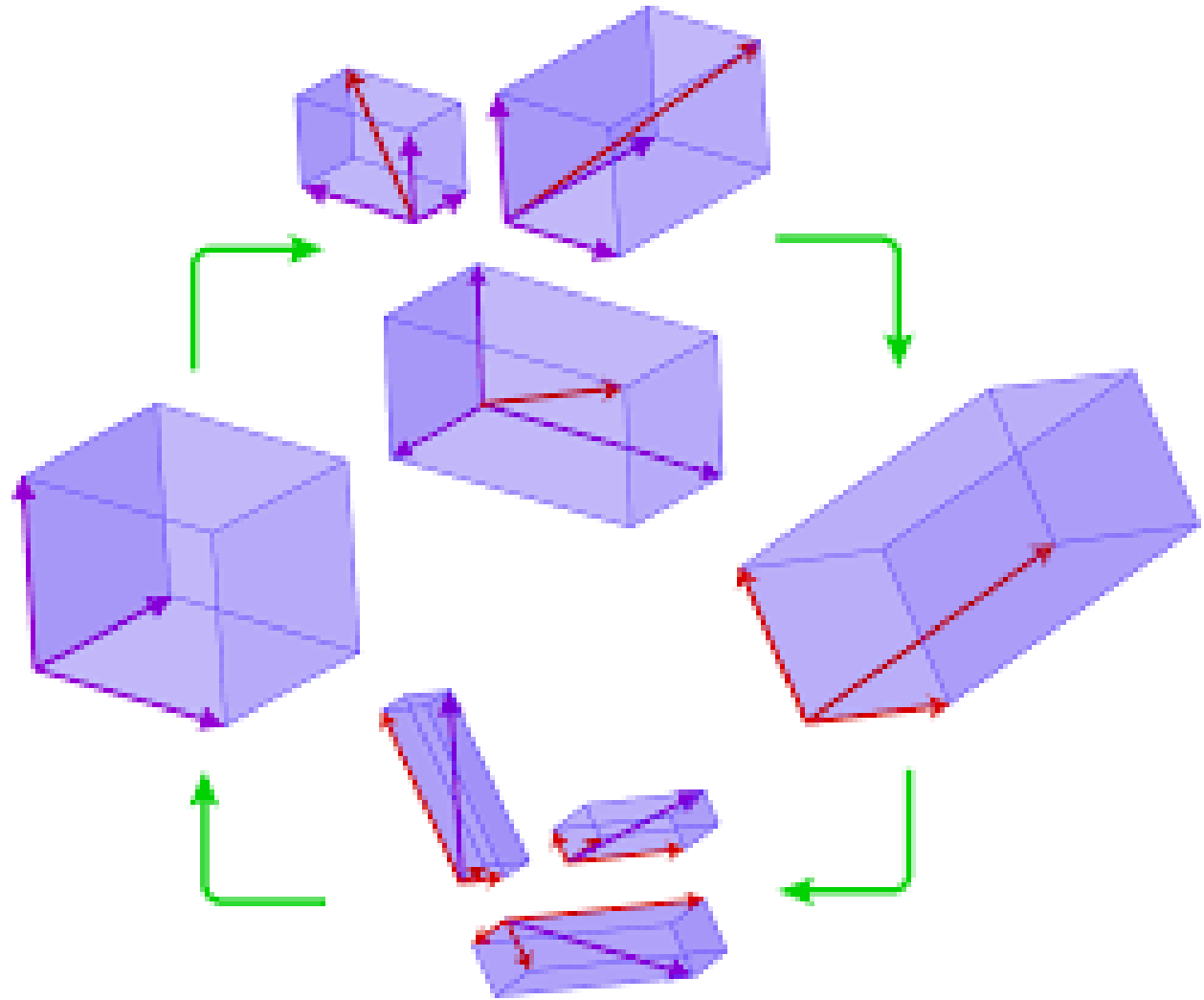
- |  |   |
|--|---|
| 1. Scaling                             | $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$                                  |
| 2. Rotation (clockwise)                | $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ |
| 3. Rotation (anti-clock)               | $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ |
| 4. Translation                         | $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$                         |
| 5. Reflection<br>(about x axis)        | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$                                     |
| 6. Reflection<br>(about y axis)        | $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$                                     |
| 7. Reflection<br>(about origin)        | $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$                                    |
| 8. Reflection about Y=X                | $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$                                      |
| 9. Reflection about Y= -X              | $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$                                    |
| 10. Shearing in X direction            | $\begin{bmatrix} 1 & 0 \\ Sh_x & 1 \end{bmatrix}$                                   |
| 11. Shearing in Y direction            | $\begin{bmatrix} 1 & Sh_y \\ 0 & 1 \end{bmatrix}$                                   |
| 12. Shearing in both x and y direction | $\begin{bmatrix} 1 & Sh_y \\ Sh_x & 1 \end{bmatrix}$                                |



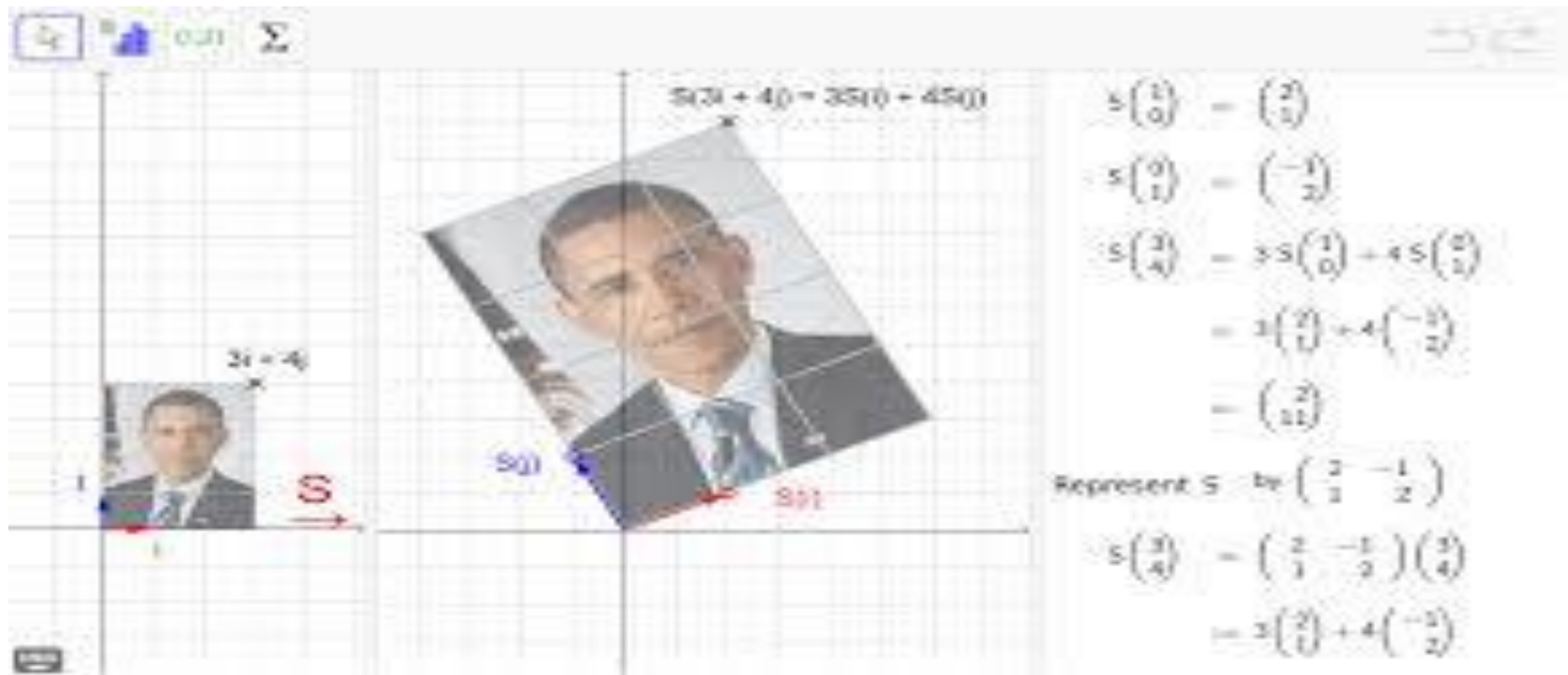
Unit vectors  
along pairwise  
mutually  
perpendicular  
standard x-, y-, z-  
axes are called  
standard basis

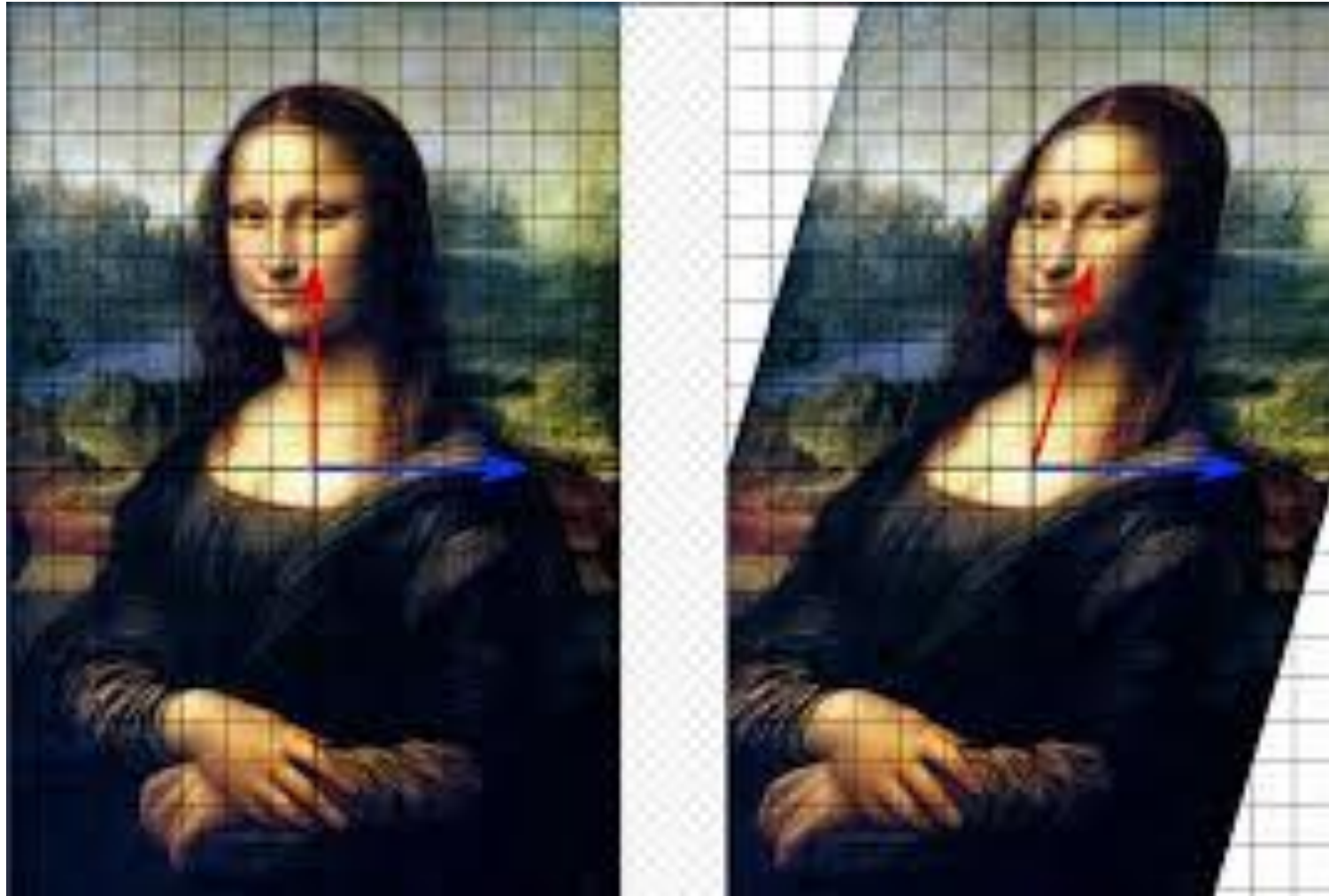


# Change of Basis



# Linear transformation

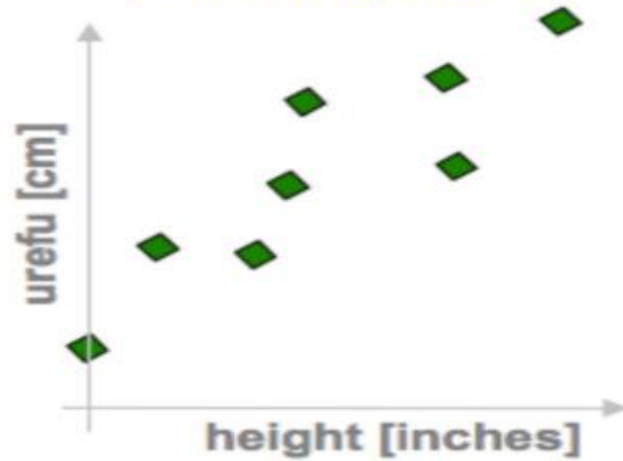




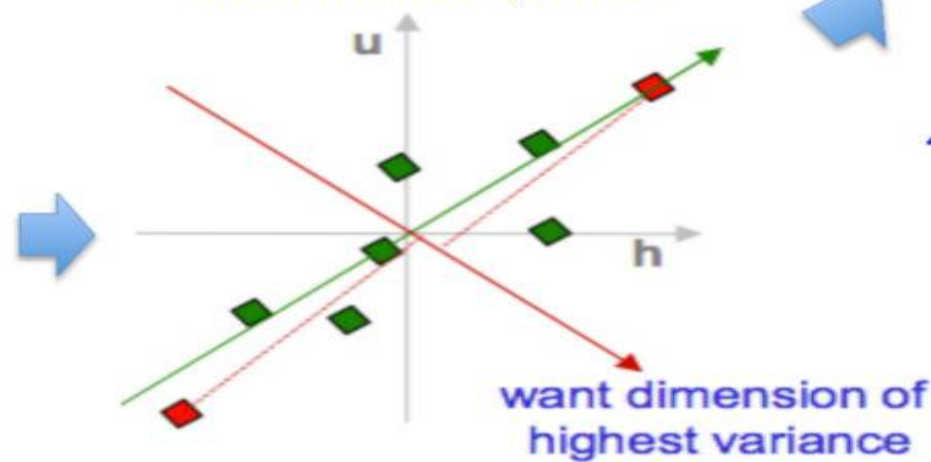
Linear transformation changes the axis too except for eigen vectors.

# PCA in a nutshell

1. correlated hi-d data  
("urefu" means "height" in Swahili)



2. center the points



3. compute covariance matrix

$$\begin{matrix} & h & u \\ h & \begin{pmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} \end{matrix} \rightarrow \text{cov}(h,u) = \frac{1}{n} \sum_{i=1}^n h_i u_i$$

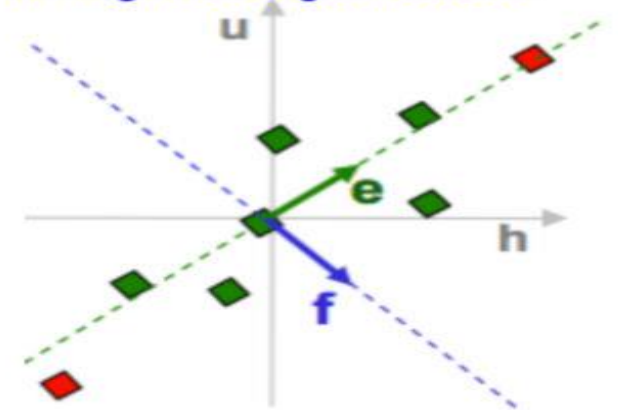
4. eigenvectors + eigenvalues

$$\begin{pmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} \begin{bmatrix} e_h \\ e_u \end{bmatrix} = \lambda_e \begin{bmatrix} e_h \\ e_u \end{bmatrix}$$

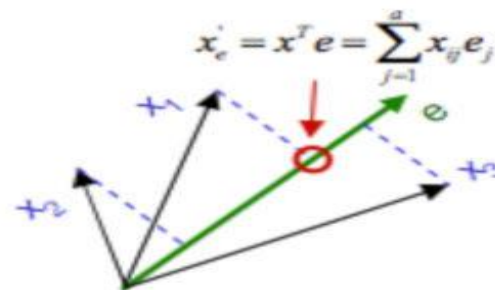
$$\begin{pmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} \begin{bmatrix} f_h \\ f_u \end{bmatrix} = \lambda_f \begin{bmatrix} f_h \\ f_u \end{bmatrix}$$

$\text{eig}(\text{cov}(\text{data}))$

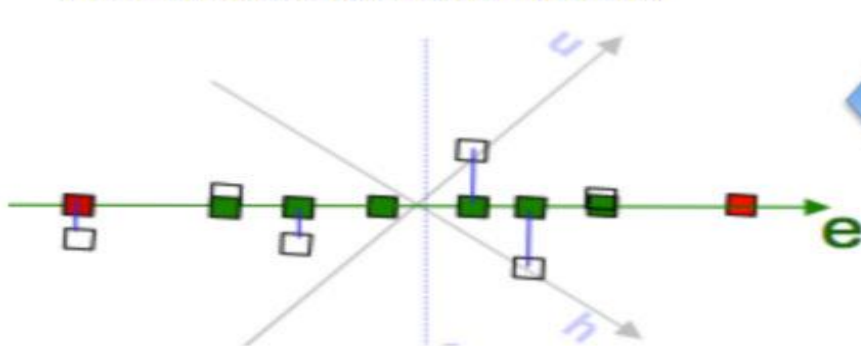
5. pick  $m < d$  eigenvectors w. highest eigenvalues



6. project data points to those eigenvectors

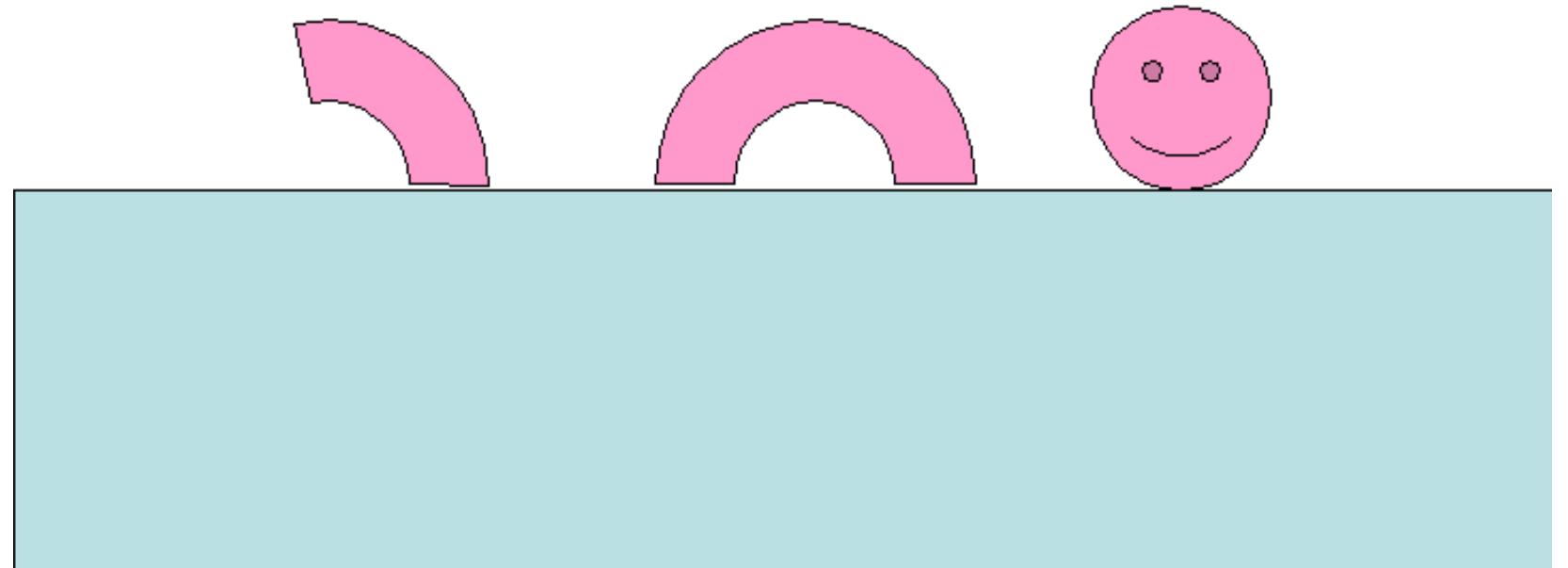


7. uncorrelated low-d data



**How many animals are under the water?**

Factor  
Analysis

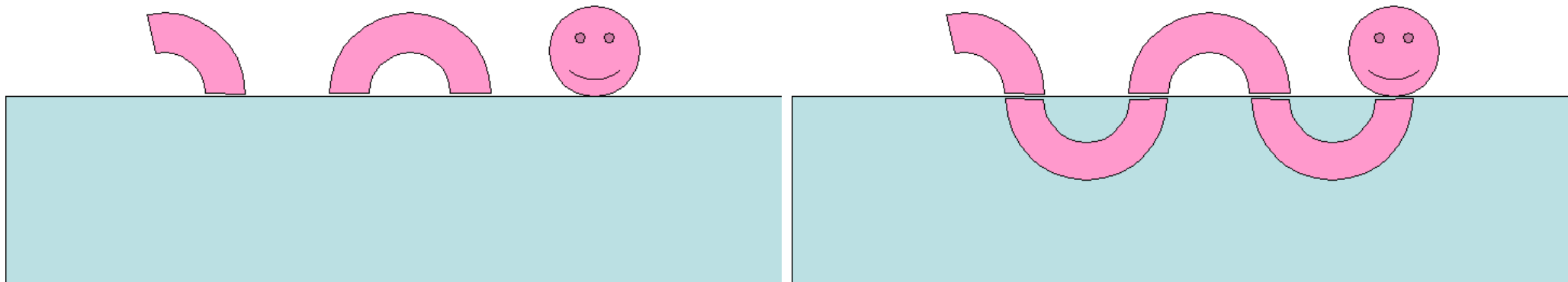




# Factor Analysis

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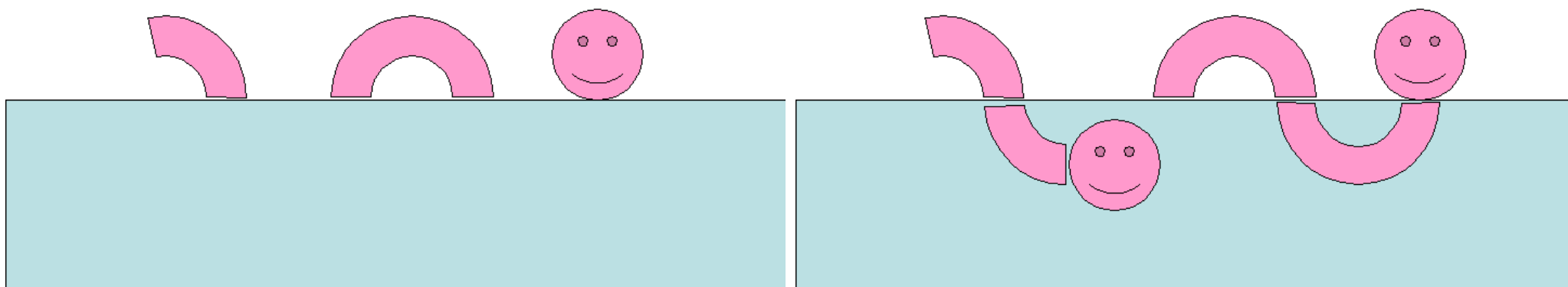
**How many animals are under the water?   How many animals are under the water?**



# Factor Analysis

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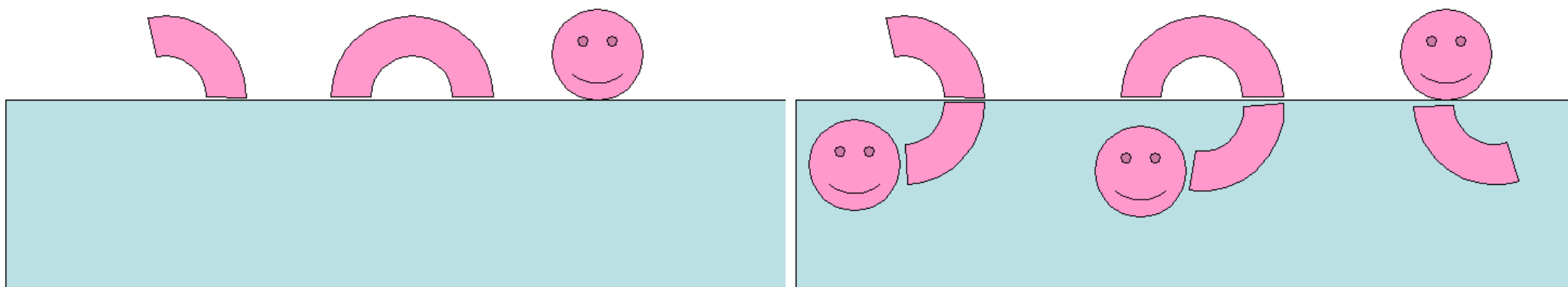
**How many animals are under the water?    How many animals are under the water?**



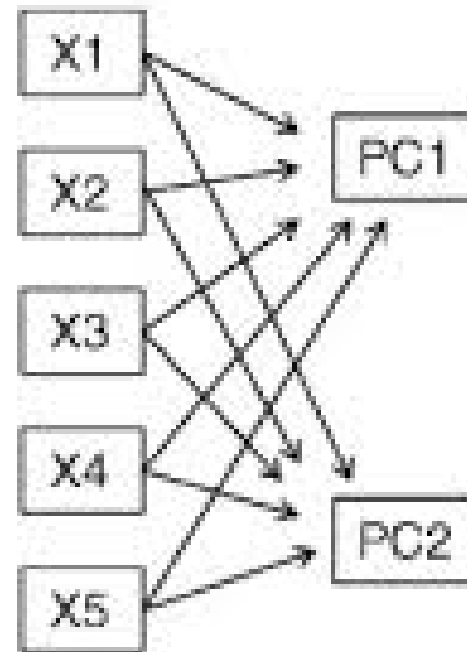
# Factor Analysis

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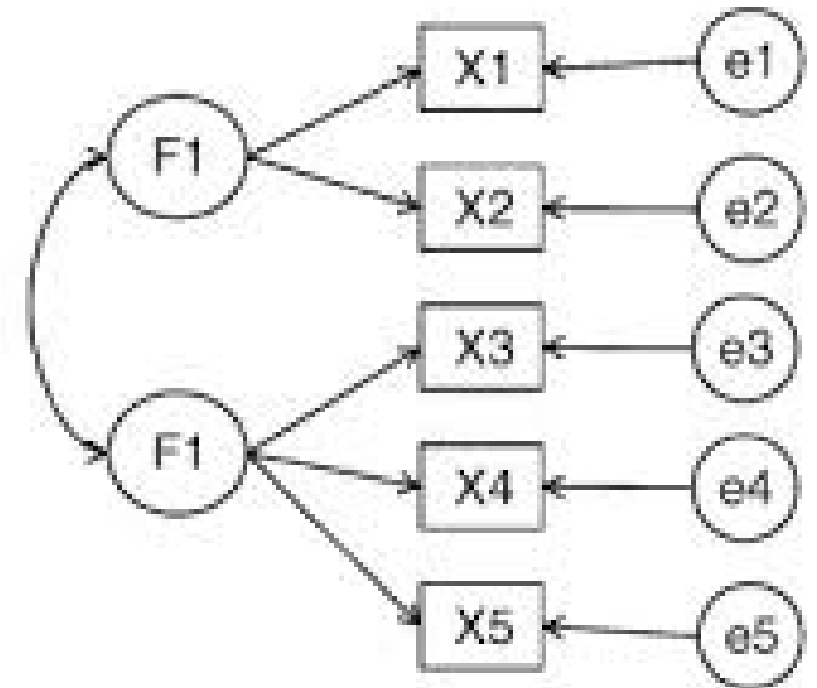
**How many animals are under the water?   How many animals are under the water?**



# PCA Vs Factor Analysis



(a) Principal Components Model



(b) Factor Analysis Model

Thank You