Mu Sigma

Thursday Learning Hour –Math Series Linear Algebra Session 2

Basics of Span, Basis, Dimension

Do The Math

Chicago, IL
Bangalore, India
www.mu-sigma.com

29th September 2022

Basic Transformation

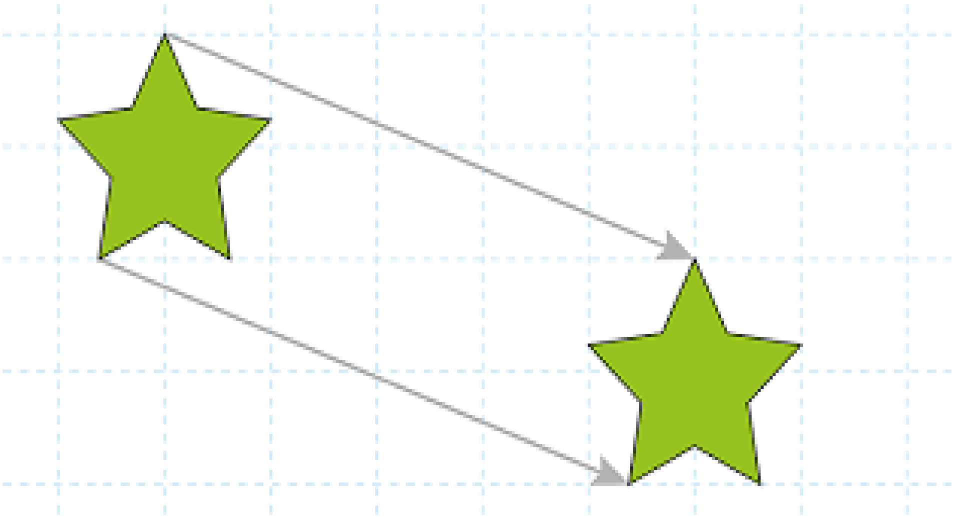
Turn	Rotate
Flip	Reflection
Slide	Translation
Resize	Dilation
Shear	Skew

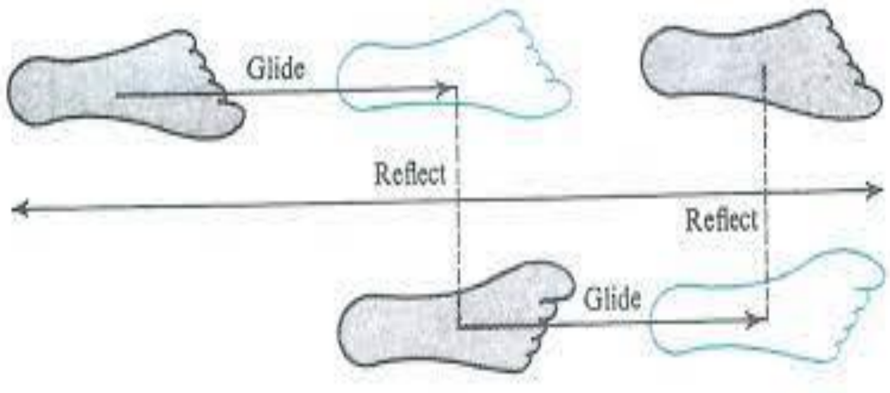
Quiz - name the geometric transformation

Quiz - name the geometric transformation

Quiz - name the geometric transformation

What is this?

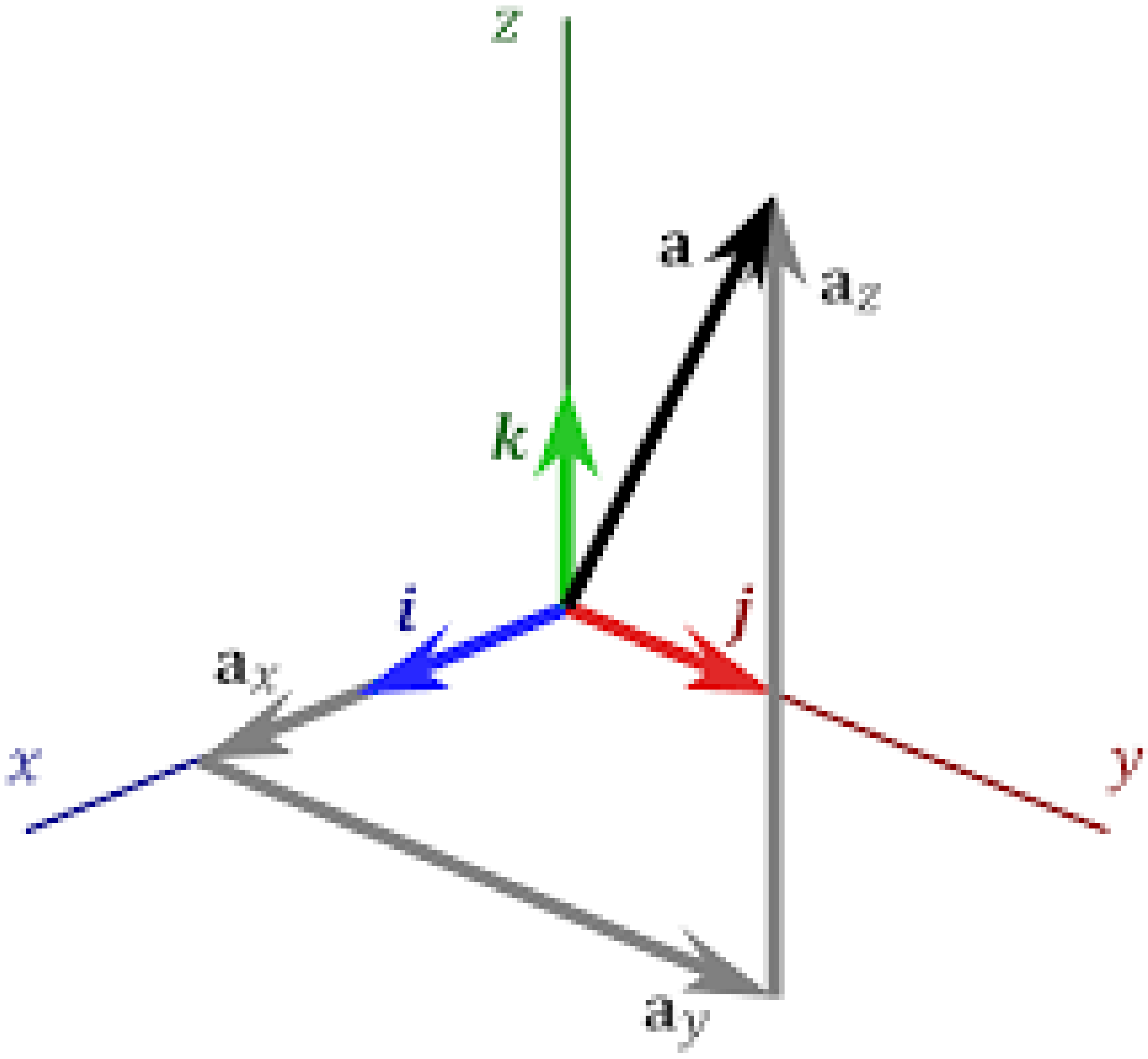




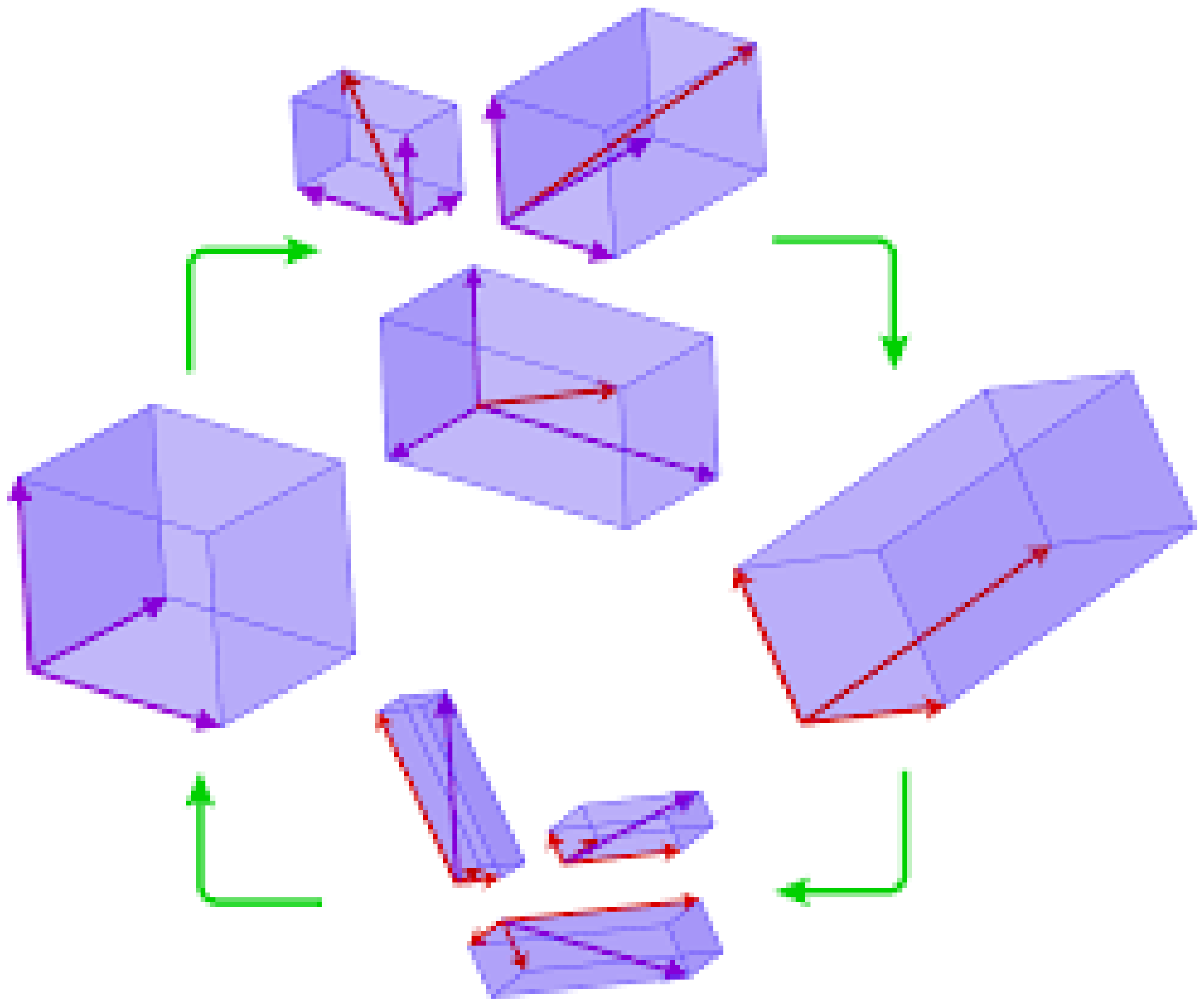
Basic transformations can be represented in a matrix form

1. Scaling $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$
2. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
3. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
4. Translation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$
5. Reflection (about x axis) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
6. Reflection (about y axis) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
7. Reflection (about origin) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
8. Reflection about Y=X $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
9. Reflection about Y= −X $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
10. Shearing in X direction $\begin{bmatrix} 1 & 0 \\ Sh_x & 1 \end{bmatrix}$
11. Shearing in Y direction $\begin{bmatrix} 1 & Sh_y \\ 0 & 1 \end{bmatrix}$
12. Shearing in both x and y direction $\begin{bmatrix} 1 & Sh_y \\ Sh_x & 1 \end{bmatrix}$

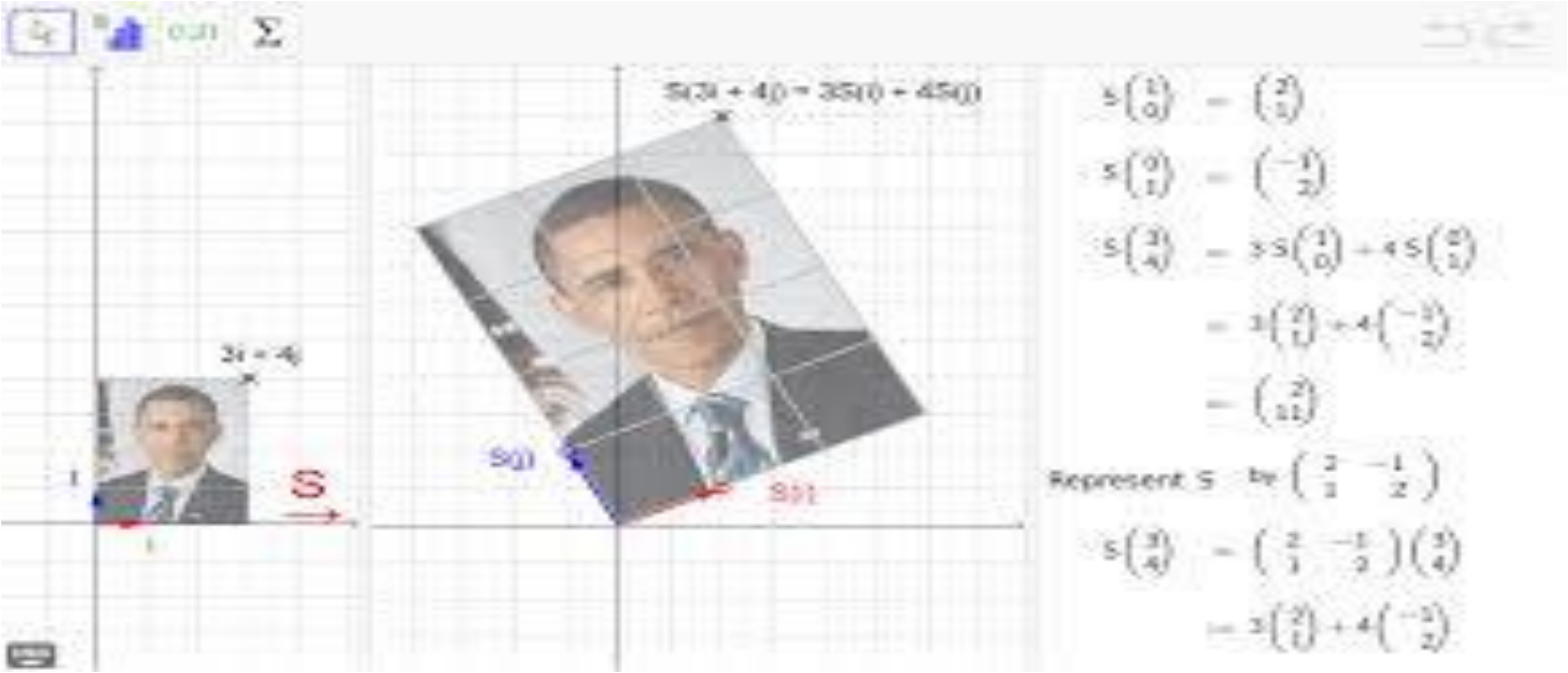
Unit vectors along pairwise mutually perpendicular standard x-, y-, z- axes are called standard basis

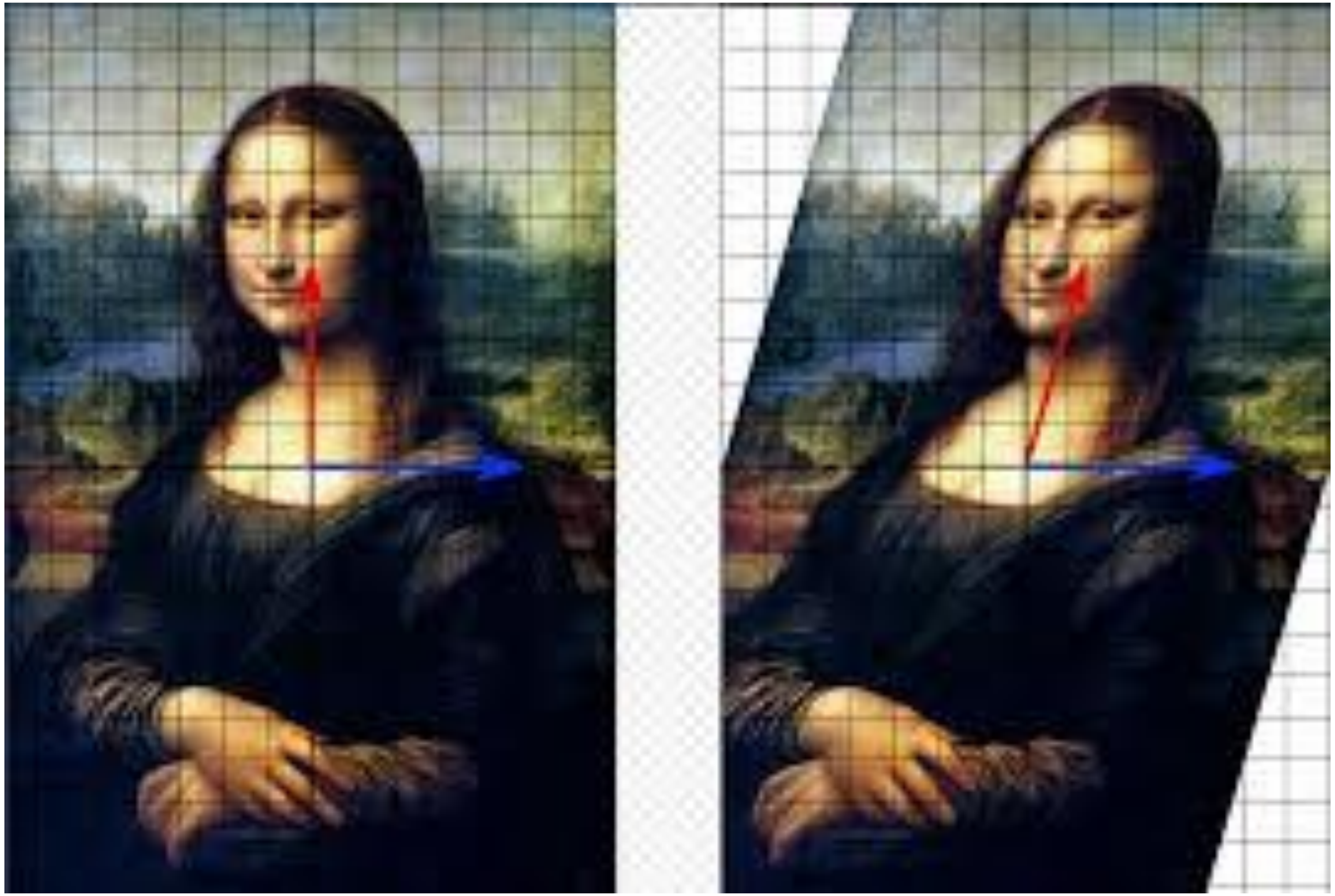


Change of Basis

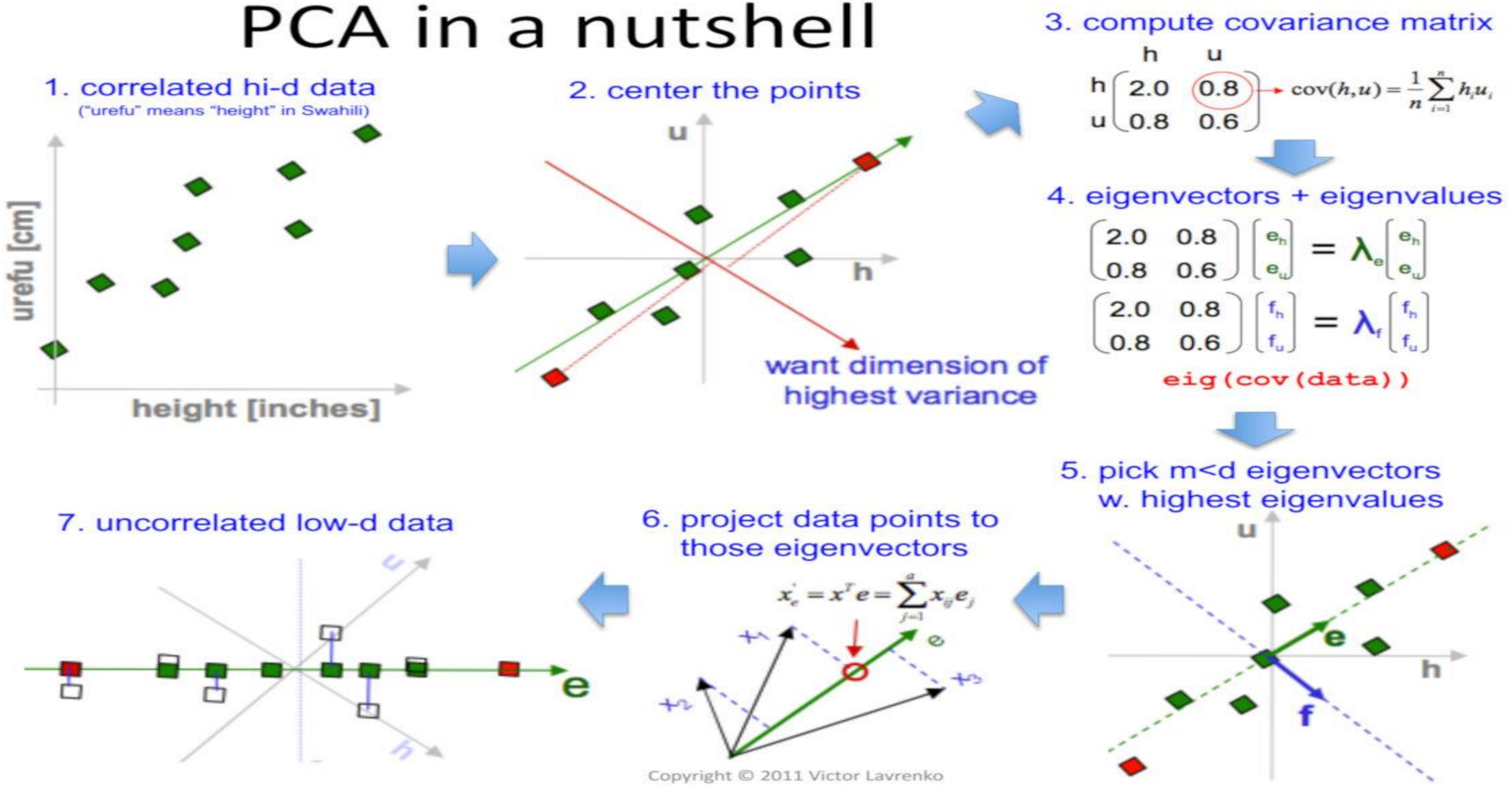


Linear transformation



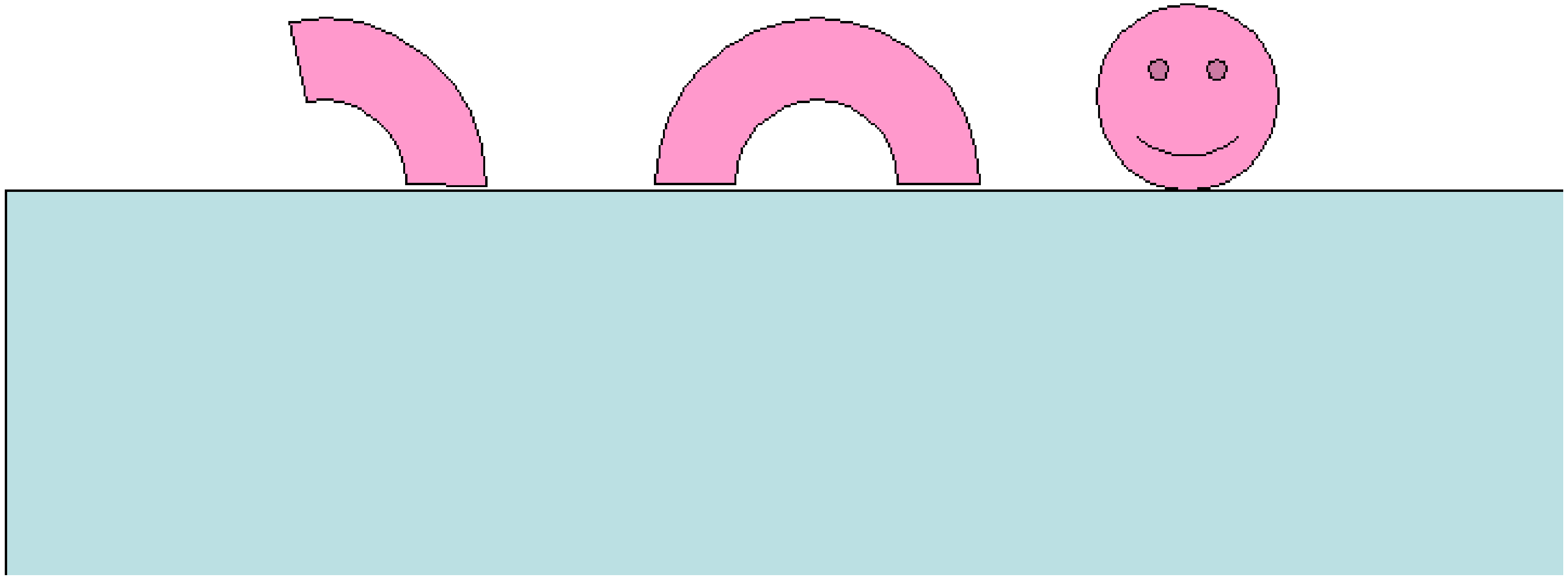


PCA in a nutshell



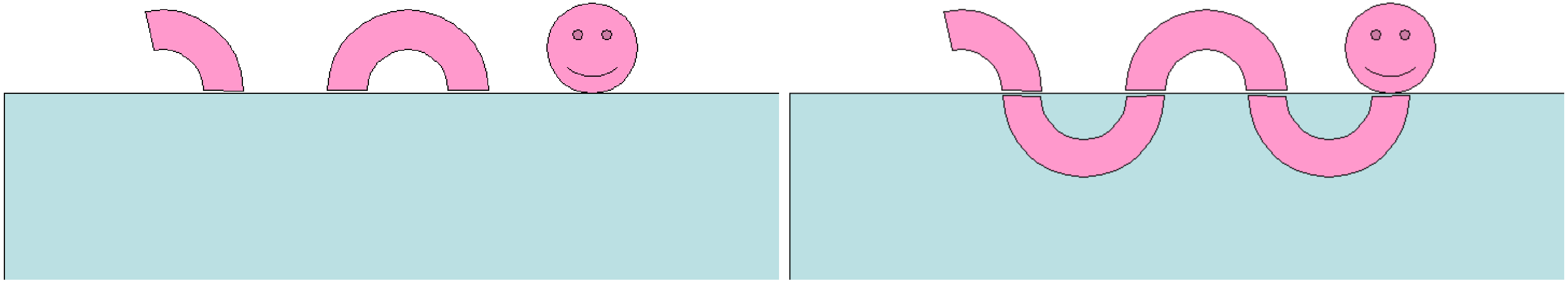
How many animals are under the water?

Factor Analysis



Factor Analysis

How many animals are under the water? How many animals are under the water?



Factor Analysis

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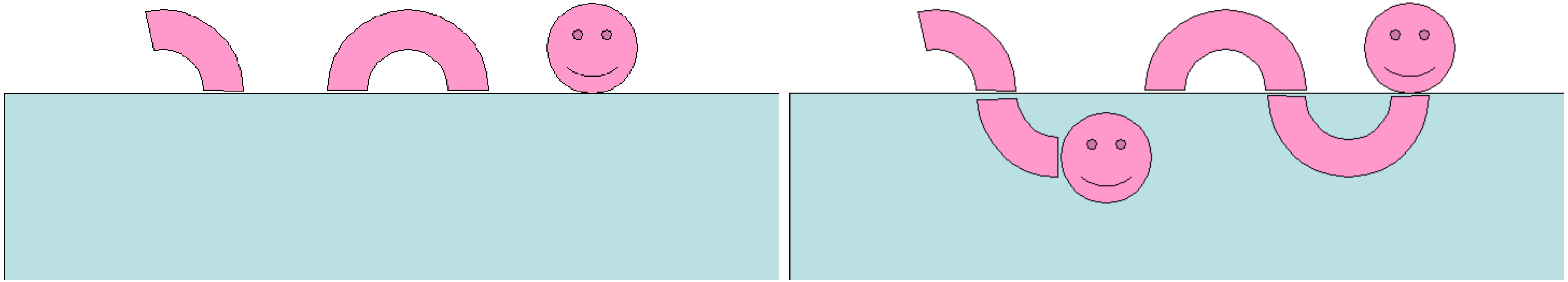

Factor Analysis

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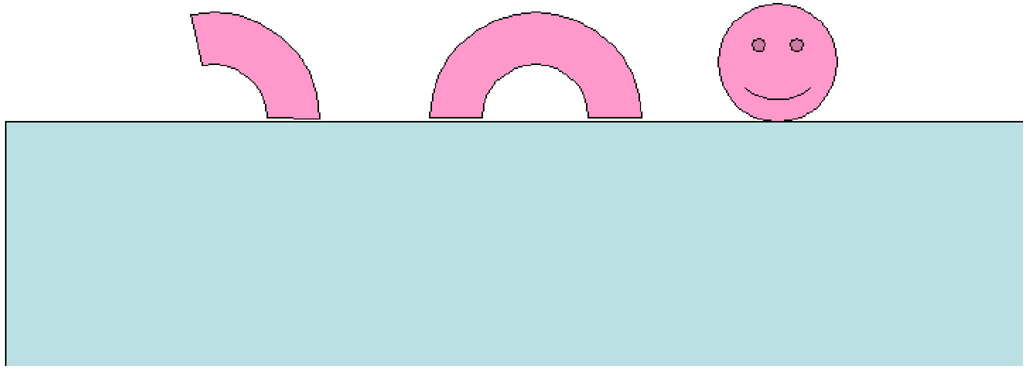

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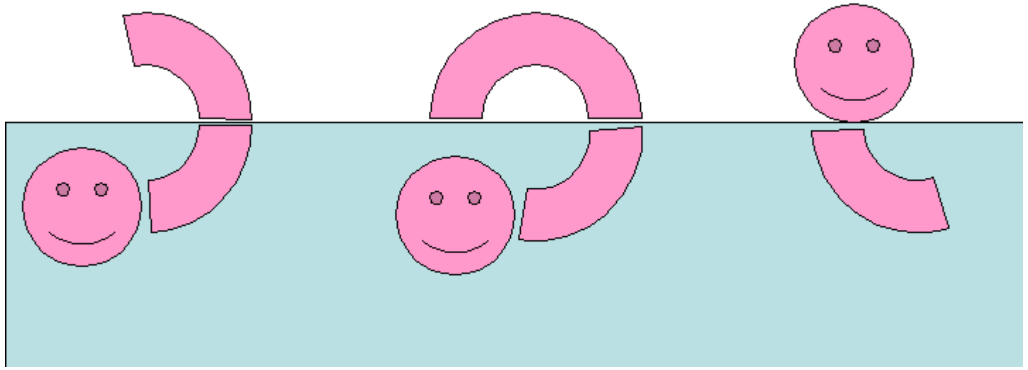
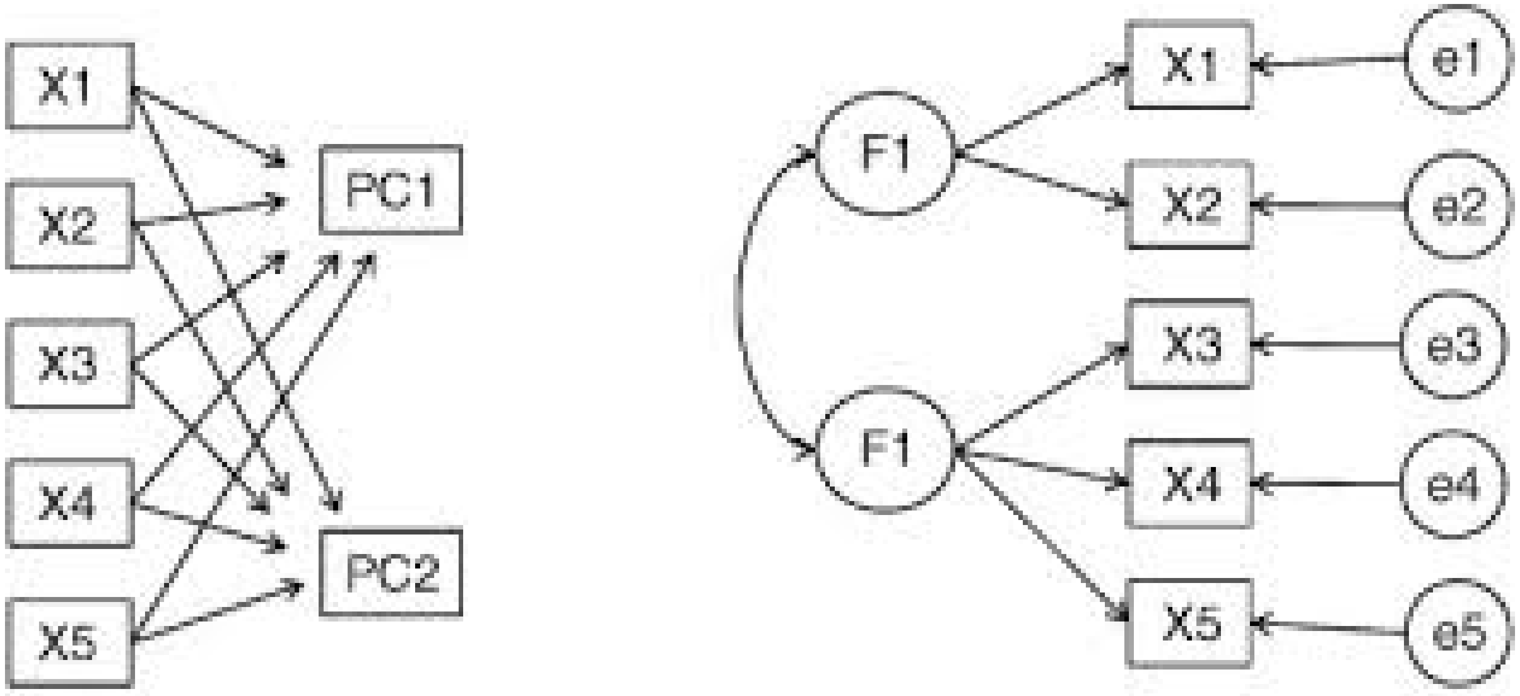


PCA Vs Factor Analysis

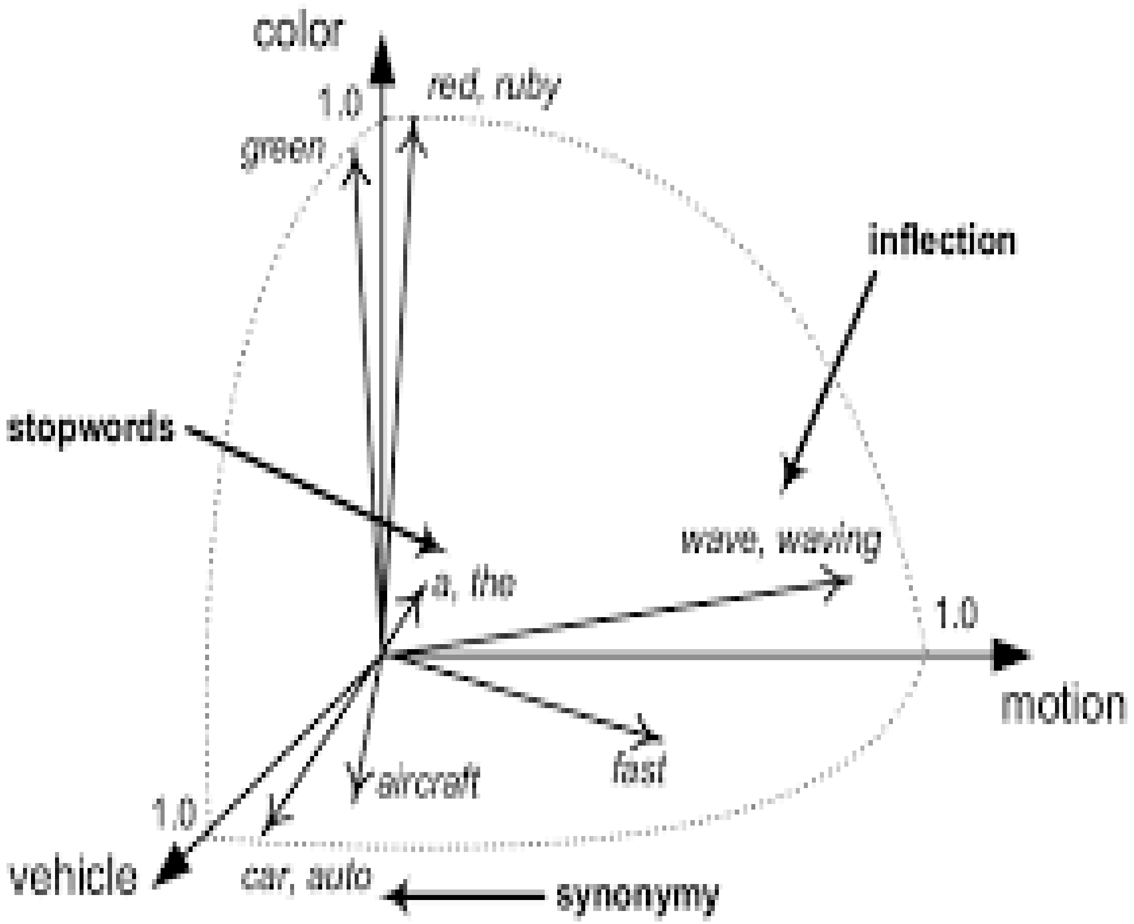


(a) Principal Components Model

(b) Factor Analysis Model

Vector space

1) $u+v$ exists in V	*closure under addition*
2) $u+v=v+u$	*communative*
3) $(u+v)+w=u+(v+w)$	*associative*
4) 0 exists in V, ie $u+0=u$	*additive identity*
5) $\forall\ u \in V\ \ \in(-u)$ s.t. $u+(-u)=0$	*inverse*
6) cu exists in V	*closure under scalar multiplication*
7) $c(u+v)\ =\ cu+cv$	*distributive*
8) $(c+d)u=cu+du$	*distributive*
9) $c(du)=(cd)u$	
10) $1u=u$	*multiplicative identity*



Gram Schmidt orthogonalization process

$$\mathbf{u}_1 = \mathbf{v}_1, \qquad \mathbf{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|}$$

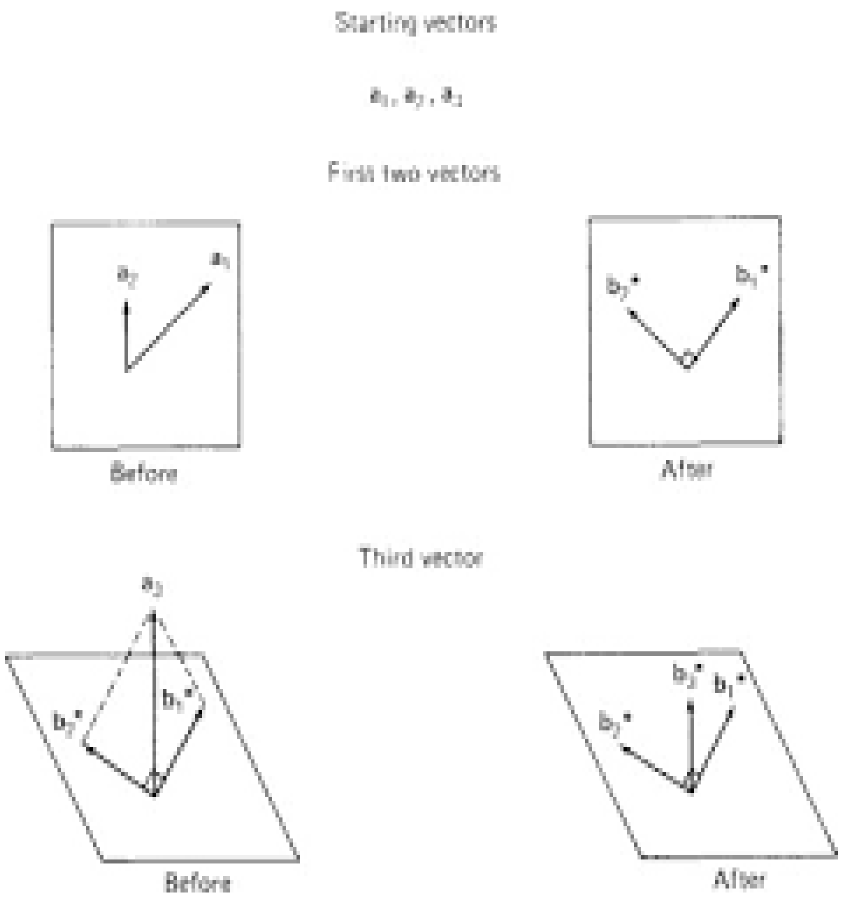
$$\mathbf{u}_2 = \mathbf{v}_2 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_2), \qquad \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|}$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_3) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_3), \qquad \mathbf{e}_3 = \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|}$$

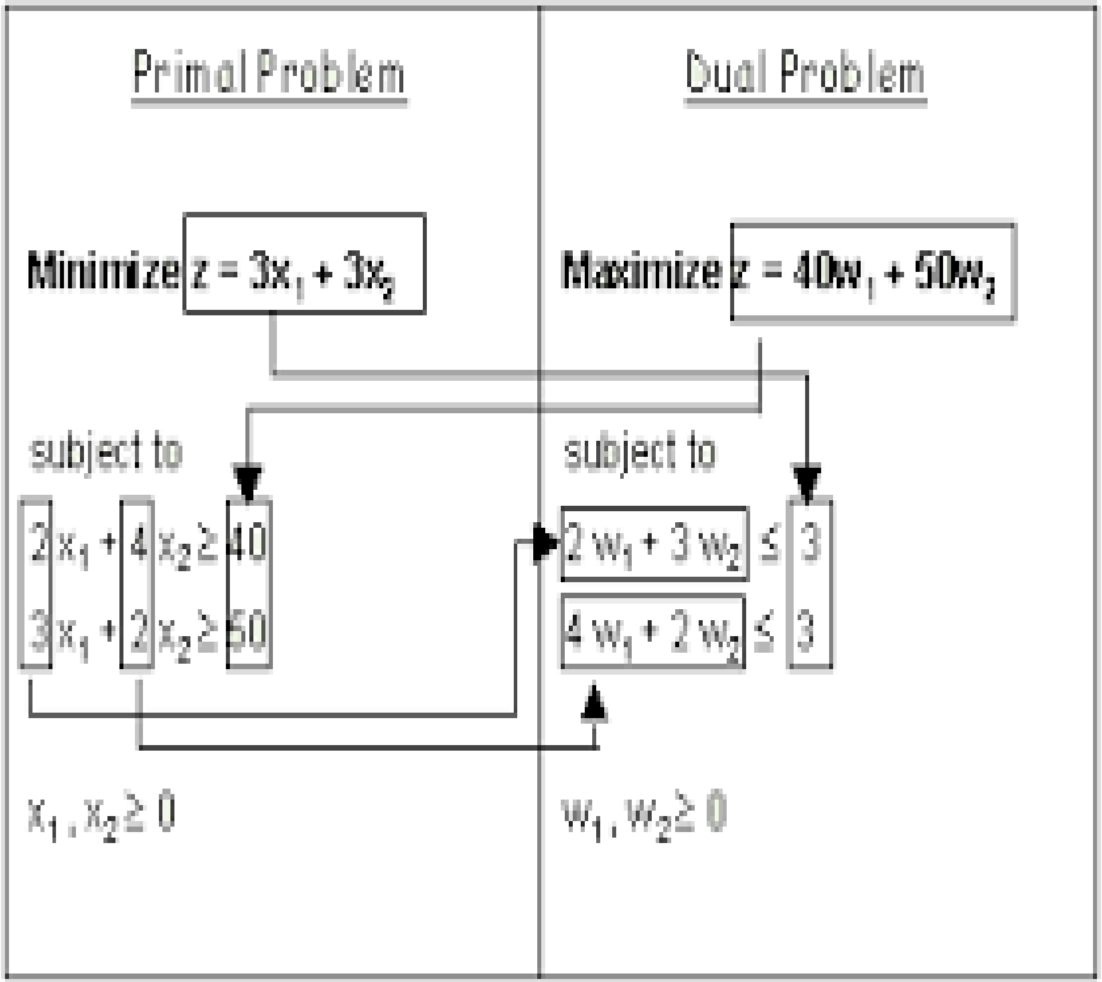
$$\mathbf{u}_4 = \mathbf{v}_4 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_3}(\mathbf{v}_4), \qquad \mathbf{e}_4 = \frac{\mathbf{u}_4}{\|\mathbf{u}_4\|}$$

$$\vdots \qquad\qquad \vdots$$

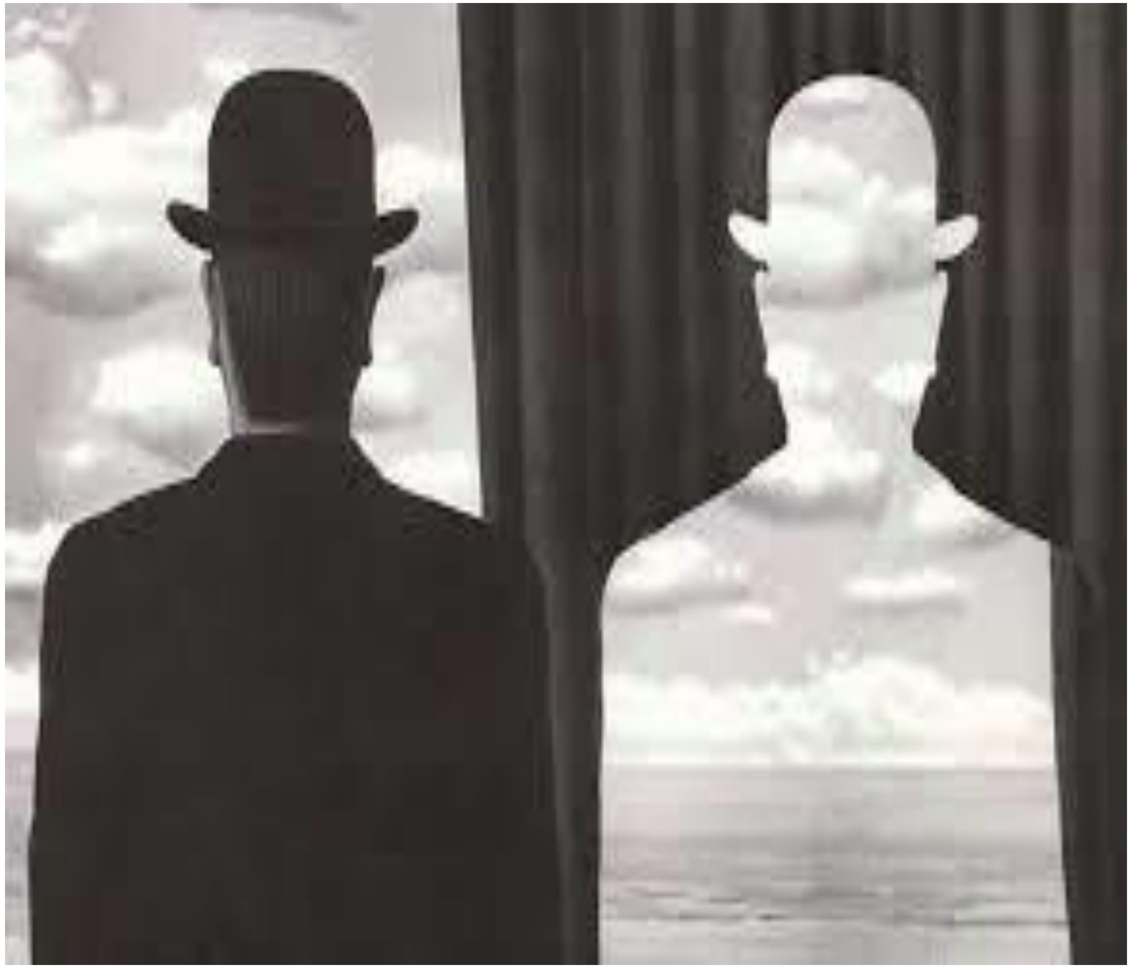
$$\mathbf{u}_k = \mathbf{v}_k - \sum_{j=1}^{k-1} \mathrm{proj}_{\mathbf{u}_j}(\mathbf{v}_k), \qquad \mathbf{e}_k = \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|},$$



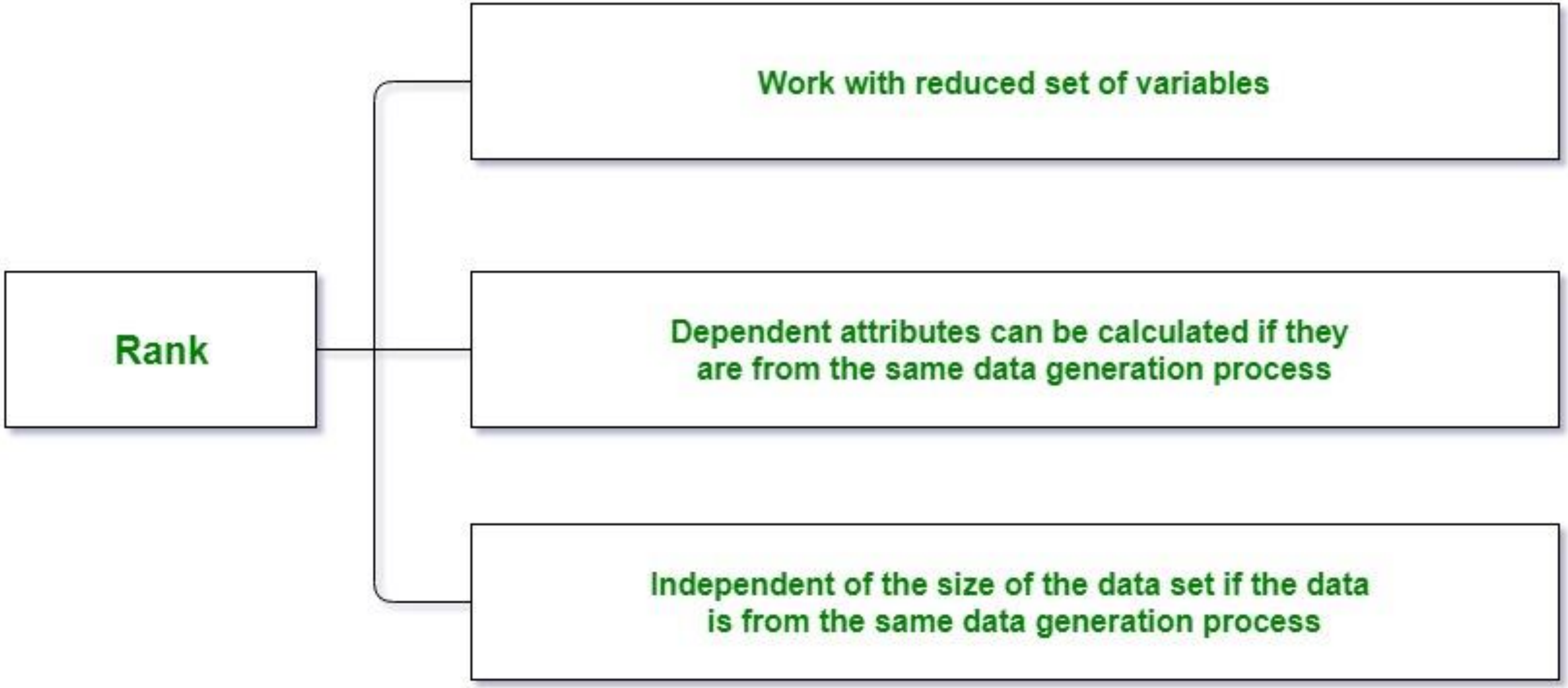
Duality



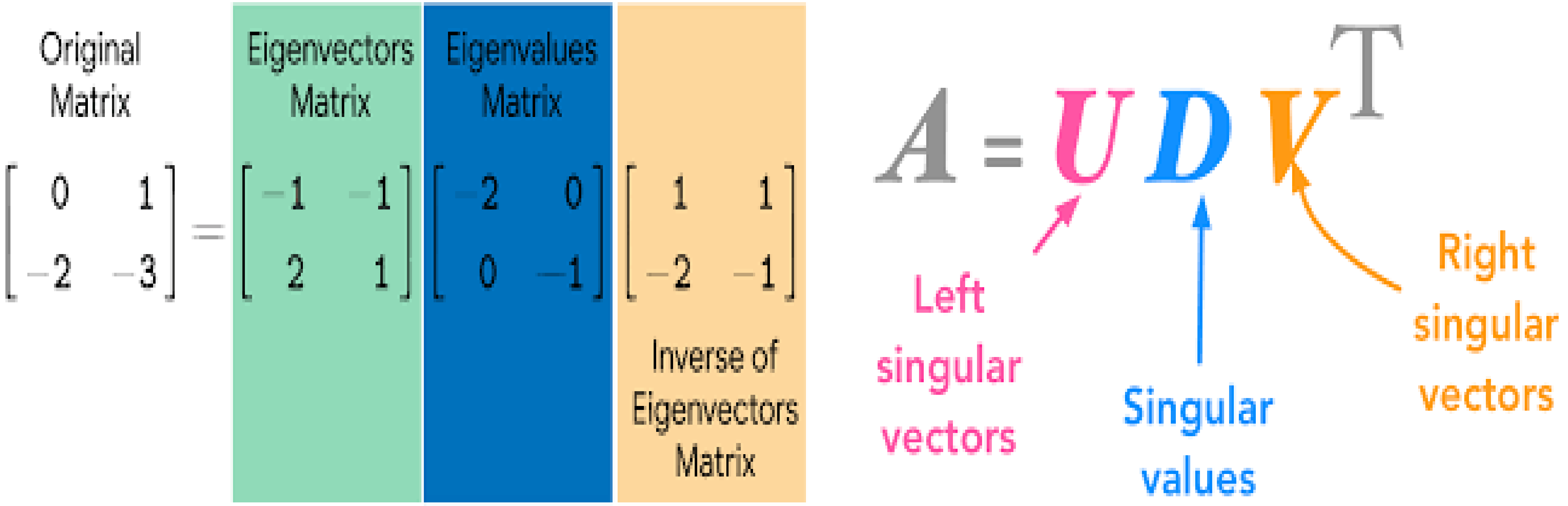
Primal Problem
Minimize $z = 3x_1 + 3x_2$
subject to
$2x_1 + 4x_2 \geq 40$
$3x_1 + 2x_2 \geq 50$
$x_1, x_2 \geq 0$
Dual Problem
Maximize $z = 40w_1 + 50w_2$
subject to
$2w_1 + 3w_2 \leq 3$
$4w_1 + 2w_2 \leq 3$
$w_1, w_2 \geq 0$



Rank


Rank
Work with reduced set of variables
Dependent attributes can be calculated if they are from the same data generation process
Independent of the size of the data set if the data is from the same data generation process

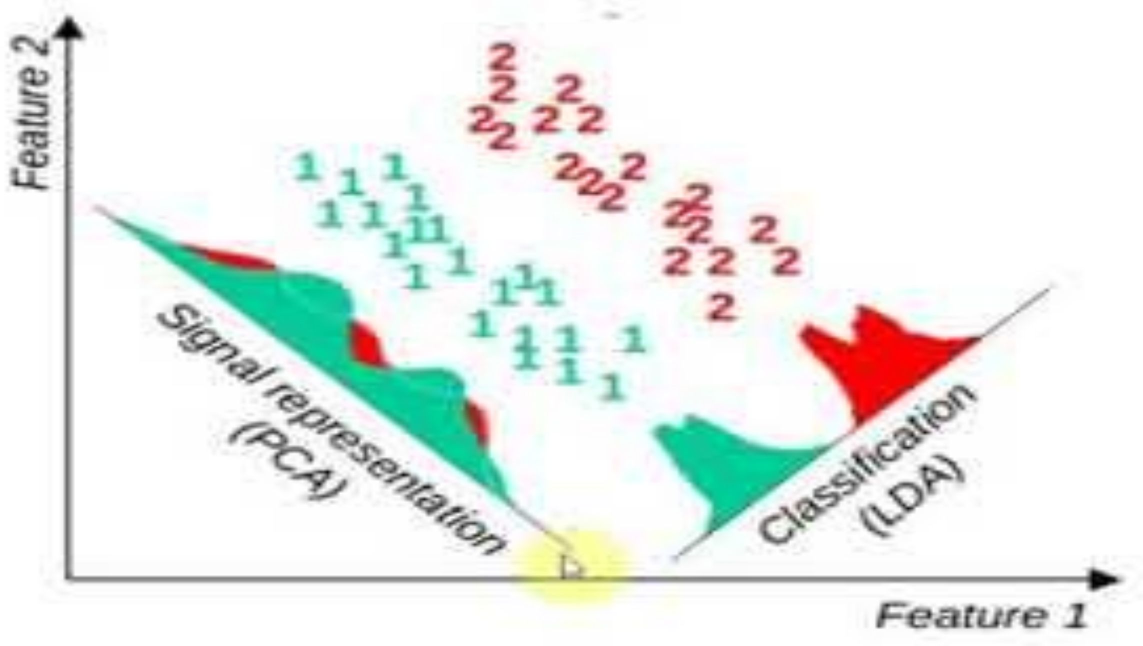
Singular value Decomposition



Difference between PCA and LDA

Quiz ?

- What is the difference between LDA & PCA?



http://stackoverflow.com/questions/33576963/dimensions-reduction-in-matlab-using-pca

Created by :- Gopal Prasad Malakar

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PCA vs LDA

Features	Principal Component Analysis	Linear Discriminant Analysis
Discrimination between classes	PCA deals with the data in its entirety for the principal components analysis without paying any particular attention to the underlying class structure.	LDA deals directly with discrimination between classes.
Supervised learning technique	PCA is an unsupervised technique.	LDA is a supervised learning technique that relies on class labels.
Focus	PCA searches for the directions that have largest variations.	LDA maximizes the ration of between-class variation and with-in class variation.
Directions of maximum discrimination	The directions of maximum variance are not necessarily the directions of the maximum discrimination since there is no attempt to use the class information such as the between-class scatter and within-class scatter	LDA is guaranteed to find the optimal discriminant directions when the class densities are Gaussian with the same covariance matrix for all the classes.
Well distributed classes in small datasets	PCA is less superior to LDA.	LDA is superior to PCA .
Computations for large datasets	PCA requires fewer computations.	LDA requires significantly more computation than PCA for large datasets
Applications	Application of PCA in the prominent field of criminal investigation is beneficial.	Linear Discriminant Analysis for data classification is applied to classification problem in speech recognition.

Figure 8: Comparison table between PCA and LDA

Thank You