Thursday Learning Hour

Monte Carlo Simulation

By

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Agenda:

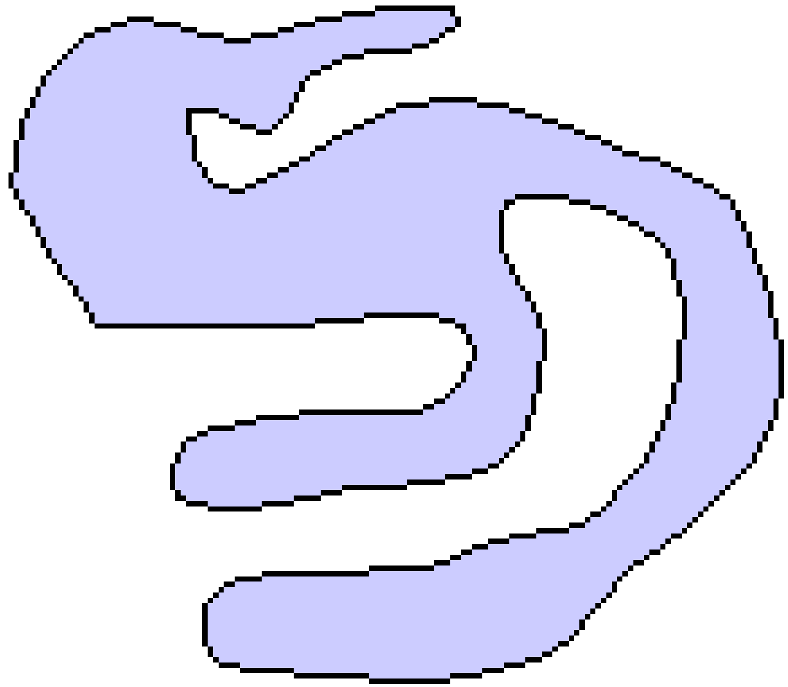
1. Idea of Monte Carlo Simulation
2. A few examples
3. Pitfalls
4. Mu Sigma artifact

Idea of randomness

Find the area of any shape:

It is very difficult to use calculus to find the area of an object such as the one shown to the right.

But using the Monte Carlo Method it's easy.



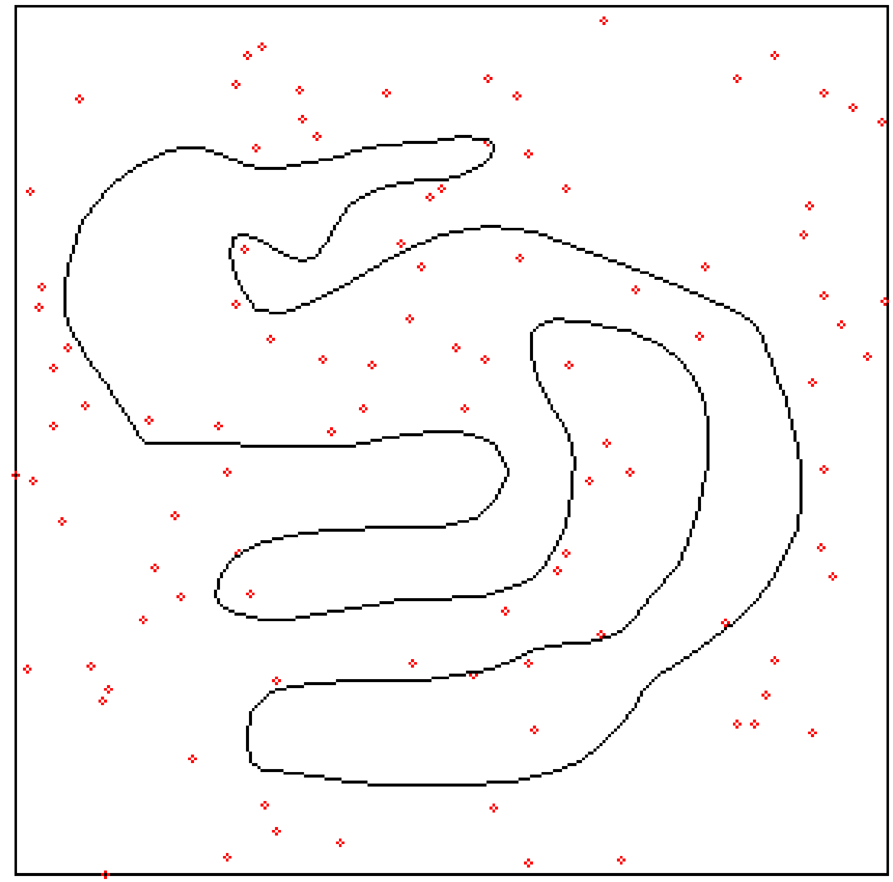
Just find the relative frequency

Box it.

Hit it randomly

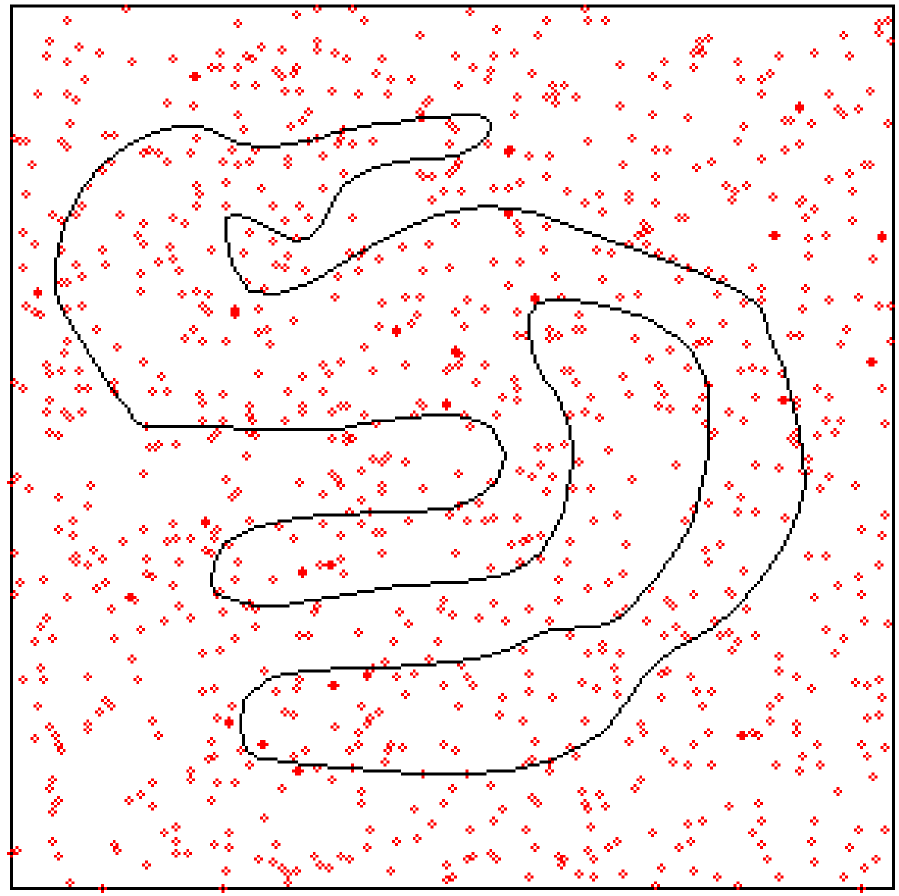
Find the proportion of hits

This proportion times the area of the box must be an indicator of the are of the image.



More trials

- Probably This is better than the previous.



Monte-Carlo Techniques

- **Problem**: What is the probability that 10 dice throws add up exactly to 32?
- **Exact Way.** Calculate this exactly by counting all possible ways of making 32 from 10 dice.
- **Approximate (Lazy) Way.** Simulate throwing the dice (say 500 times), count the number of times the results add up to 32, and divide this by 500.
- **Lazy Way can get quite close to the correct answer quite quickly.**

Monte Carlo simulation: technique that combines ***distributions*** with ***random number generation***

Random numbers can be generated in different ways

Any variable has a probability distribution for its occurrence

Best way to relate random number to a variable is to use **cumulative probability distribution**

(probability density functions – pdf)

The daily demand for clone packs (80 seedlings) during Spring months was studied and the probabilities are the following:

Relative frequencies (probability) → If the distribution is known , **WHY** do we use **random numbers** to simulate it?

↓

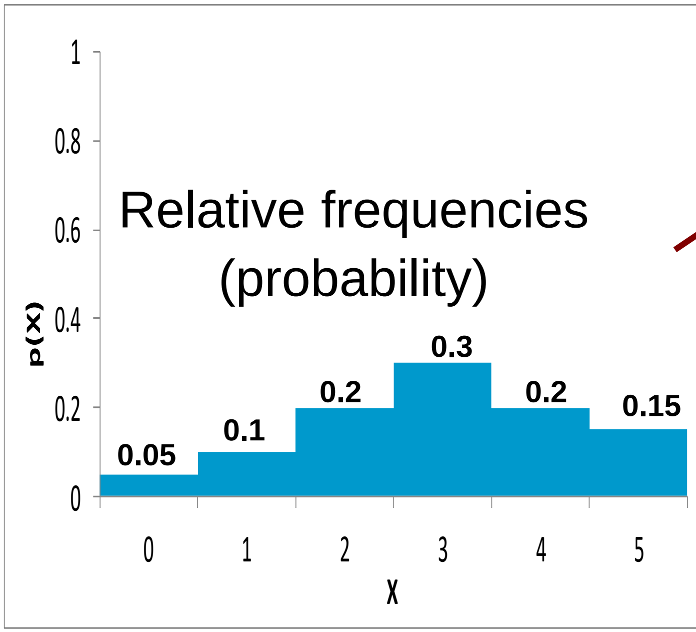
BECAUSE, although the **probability is known** (the relative frequency of each demand level), **the order of occurrence is not**

↓

It is **the order of occurrence** (which is assumed random) **which we want to simulate**

Nr packs ordered	probability
0	0.05
1	0.1
2	0.15
3	0.3
4	0.25
5	0.15

Assume that the demand/day is given by:



If the distribution is known , **WHY** do we use **random numbers** to simulate it?

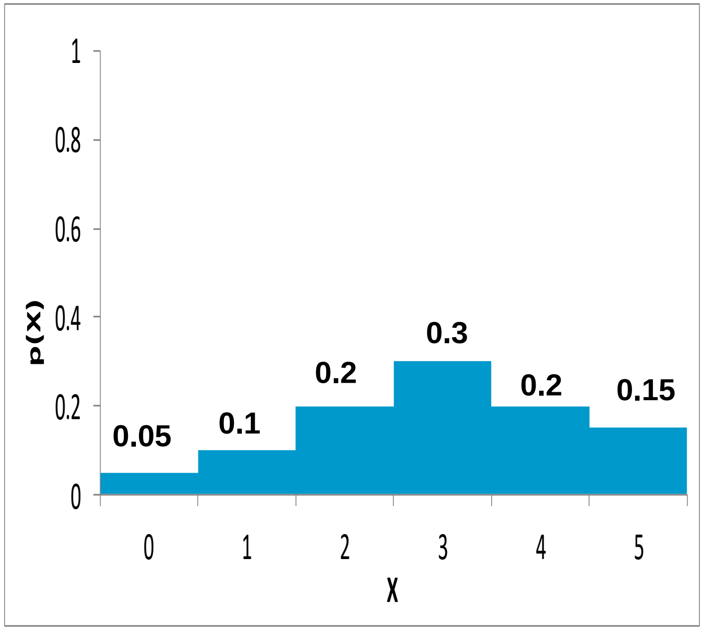
BECAUSE, although the **probability is known** (the relative frequency of each demand level), **the order of occurrence is not**

It is **the order of occurrence** (which is assumed random) **which we want to simulate**

Sample: a proper subset of the population

Assume that the demand/day is given by:

Relative frequencies (probability)'

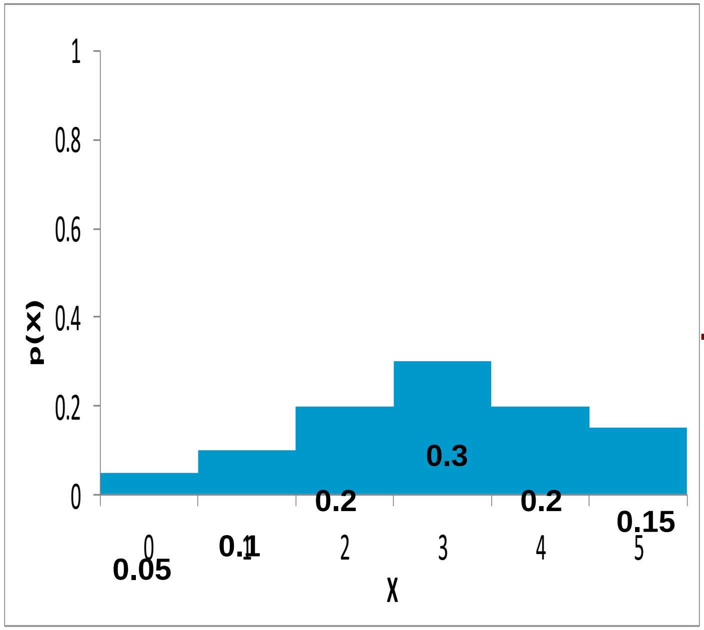


If 10000 **random numbers** were drawn it would be expected that the number of observations per class would be:

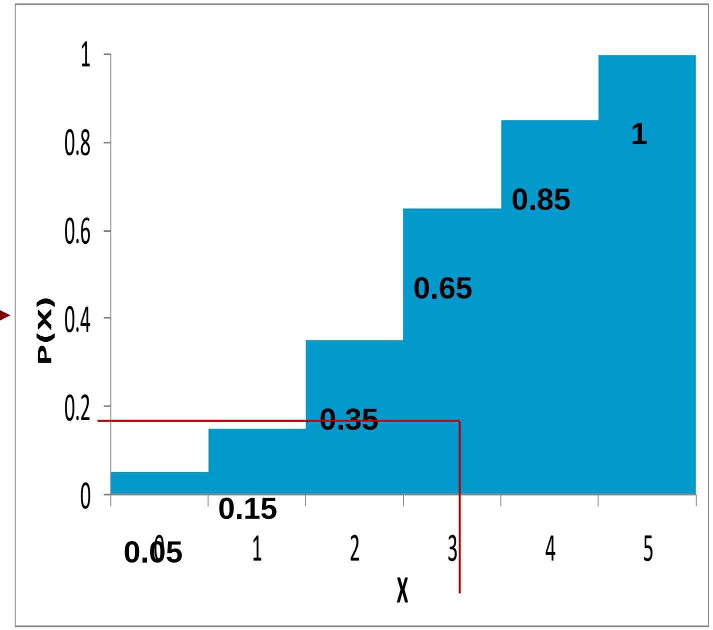
Demand (x)	frequencies	observations
0	0.05	500
1	0.1	1000
2	0.2	2000
3	0.3	3000
4	0.2	2000
5	0.15	1500

Assume that the demand/day is given by:

Relative frequencies (probability)



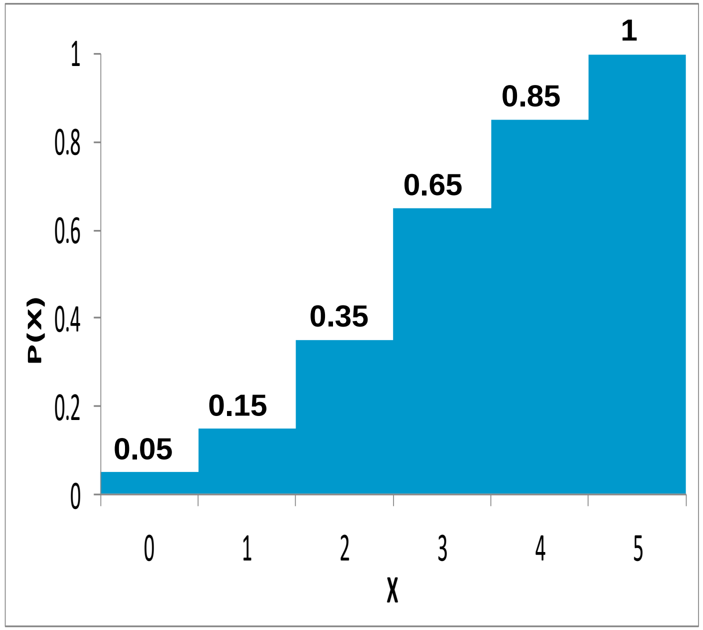
Cumulative frequencies (probability)



Assume that the demand/day is given by:

Demand (x)	Cumulative probability	Interval for random numbers
0	0.05	0 – 4
1	0.15	5 – 14
2	0.35	15 – 34
3	0.65	35 – 64
4	0.85	65 – 84
5	1	85 – 99

Cumulative frequencies (probability)



Assume that the demand/day is given by:

Simulate the demand for 10 days

Demand (x)	Cumulative frequencies	Interval for random numbers
0	0.05	0 – 4
1	0.15	5 – 14
2	0.35	15 – 34
3	0.65	35 – 64
4	0.85	65 – 84
5	1	85 – 100

day	Random number	demand
1	14	1
2		
3		
4		
5		
6		
7		
8		
9		
10		

Assume that the demand/day is given by:

Demand (x)	Cumulative frequencies	Interval for random numbers
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4	0.85	65 – 84
5	1	85 – 100

Simulate the demand for 10 days

day	Random number	demand
1	14	1
2	74	4
3		
4		
5		
6		
7		
8		
9		
10		

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5	1	85 – 100

Simulate the demand for 10 days

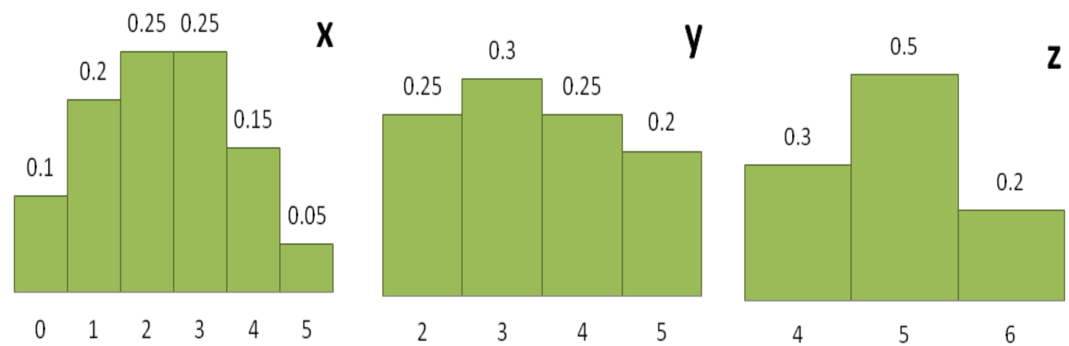
day	Random number	demand
1	14	1
2	74	4
3	24	2
4	87	5
5	7	1
6	45	3
7	26	2
8	66	4
9	26	2
10	94	5

In general, Monte Carlo Simulation is roughly composed of five steps:

1. Set up probability distribution that will be considered in the simulation
2. Build cumulative probability distribution
3. Establish an interval of random numbers for each variable
4. Generate random numbers
5. Simulate trials

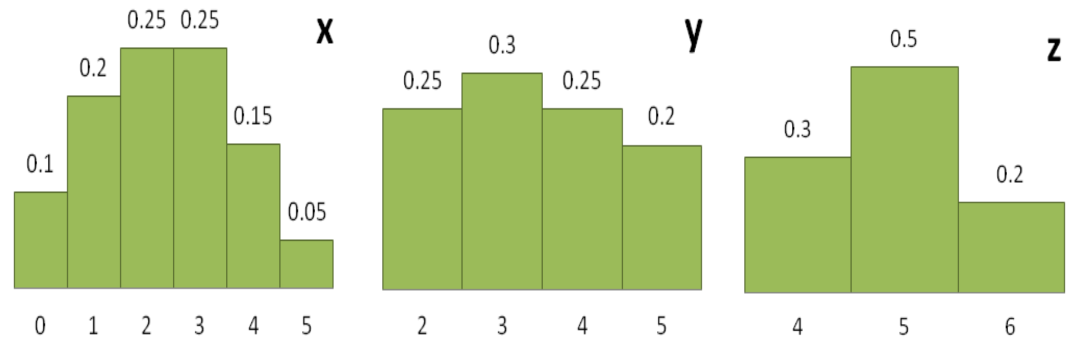
Example 2 - Assume the effectiveness function for a system is W = 5x + 2y + z, where the variables x, y and z are independent and described by the probabilities below. Run 18 trials

1. Set up probability distributions that will be considered in the simulation

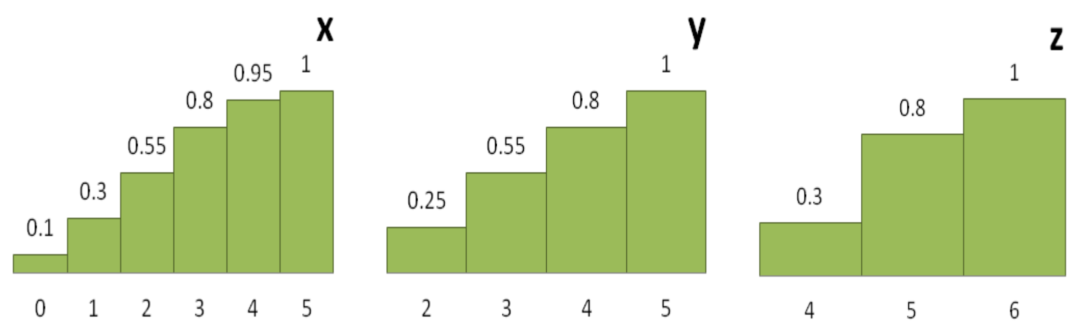


Example 2 - **Assume the effectiveness function for a system is W = 5x + 2y + z, where the variables x, y and z are independent and described by the probabilities below. Run 18 trials**

1. Set up probability distributions that will be considered in the simulation



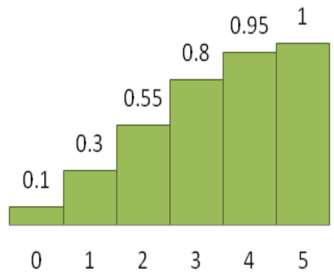
2. Build cumulative probability distributions



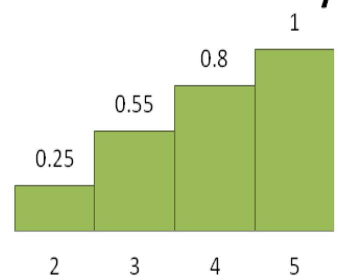
Example 2 - Assume the effectiveness function for a system is W = 5x + 2y + z, where the variables x, y and z are independent and described by the probabilities below. Run 18 trials

1. Set up probability distributions that will be considered in the simulation
2. Build cumulative probability distributions

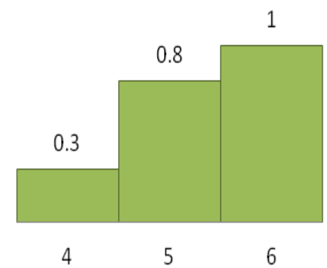
x



y



z



3. Establish an interval of random numbers for each variable

x	distribution	cumulative distribution	aux	lower lim	upper lim	interval
0	0.1	0.1	10	0	9	0 - 9
1	0.2	0.3	30	10	29	10 - 29
2	0.25	0.55	55	30	54	30 - 54
3	0.25	0.8	80	55	79	55 - 79
4	0.15	0.95	95	80	94	80 - 94
5	0.05	1	100	95	99	95 - 99
y	distribution	cumulative distribution	aux	lower lim	upper lim	interval
2	0.25	0.25	25	0	24	0 - 24
3	0.3	0.55	55	25	54	25 - 54
4	0.25	0.8	80	55	79	55 - 79
5	0.2	1	100	80	99	80 - 99
z	distribution	cumulative distribution	aux	lower lim	upper lim	interval
4	0.3	0.3	30	0	29	0 - 29
5	0.5	0.8	80	30	79	30 - 79
6	0.2	1	100	80	99	80 - 99

Example 2 - Assume the effectiveness function for a system is W = 5x + 2y + z, where the variables x, y and z are independent and described by the probabilities below. Run 18 trials

1. Set up probability distributions that will be considered in the simulation
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trial nr	rand_x	rand_y	rand_z
1	43	22	1
2	74	9	8
3	84	10	82
4	42	38	65
5	83	16	34
6	25	1	27
7	21	67	62
8	25	38	58
9	83	65	42
10	76	25	32
11	74	27	63
12	68	73	55
13	3	7	96
14	60	53	29
15	35	34	31
16	56	25	17
17	71	83	83
18	15	72	49

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4	42	38	65	2	3	5	21
5	83	16	34	4	2	5	29
6	25	1	27	1	2	4	13
7	21	67	62	1	4	5	18
8	25	38	58	1	3	5	16
9	83	65	42	4	4	5	33
10	76	25	32	3	3	5	26
11	74	27	63	3	3	5	26
12	68	73	55	3	4	5	28
13	3	7	96	0	2	6	10
14	60	53	29	3	3	4	25
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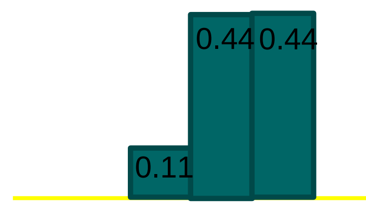
18	15	72	49	1	4	5	18

class	freq	prob
5 – 14 (10)	2	(2/18) 0.11
15 – 24 (20)	8	(8/18) 0.44
25 – 34 (30)	8	(8/18) 0.44
35 – 44 (40)	0	0
	18	



Mu Sigma Artifact

Thank You